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ANOMALOUS CROSS SECTION FOR PHOTON-PHOTON SCATTERING IN GAUGE THEORIES *

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We consider the deep inelastic structure functions of the photon in an asymptotically free gauge theory. In contrast to the case of a hadronic target, we find that the short-distance analysis determines the shape and magnitude and not merely the Q^2 dependence of the structure functions. The structure functions of the free quark theory are renormalized by finite, calculable factors. For example, at $x = 0.1$, we find that F_2 will, at large Q^2, exceed the free quark result by a factor 1.751, while for $x = 0.5$, F_2 is suppressed asymptotically, relative to the free quark theory, by a factor 0.964, and at $x = 0.8$, by a factor 0.611.

1. Introduction

The short distance analysis of deep inelastic scattering [1] makes the following predictions: The scaling behavior of the structure functions, for any hadron target, differs from that of free field theory by known logarithmic factors; the actual shape of the structure functions is not determined. Here we consider the corresponding problem with a photon target; in other words, we are considering the total cross section for $\gamma + \gamma \to$ hadrons, with one photon on or near mass shell while the other photon has a large Euclidean momentum. It has been suggested [2] that this cross section could be measured by studying $e^{\pm}e^{\pm} \to e^{\pm}e^{\pm}$ + hadrons in colliding beams. We find the following, somewhat anomalous results: The scaling behavior is that of the free quark model; the magnitude and shape of the structure functions are completely determined, but do *not* agree with the free quark model.

Technically, these results arise as follows. Because the photon is an elementary particle, there are new "contact" terms in the short distance analysis. Because the photon interacts weakly, the new terms can be explicitly evaluated. Moreover, they turn out to be the dominant terms for large Q^2. If there are other weakly interacting

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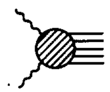
I 

Fig. 1. The total cross section $\gamma + \gamma \to$ hadrons.

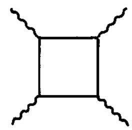
II 

Fig. 2. The box diagram.

elementary particles in nature, such as elementary Higgs bosons, we expect similar results for scattering from such targets.

That there are extra contact terms in the short distance analysis with a photon target is already known from work of Walsh and Zerwas and of Kingsley [3]. The photon structure functions were recently discussed by Ahmed and Ross within the context of gauge theories [4].

In the free quark model, the photon structure functions can be calculated by evaluating the s-channel discontinuity of the box diagram of fig. 2. One obtains

$$F_2(x) = \frac{\sum e_i^4}{16\pi^2}\left[x(1-2x(1-x))\ln\left(\frac{Q^2}{m_{\mathrm{q}}^2}\,\frac{1}{x}-1\right) - x + 8x^2(1-x)\right],$$

$$F_{\mathrm{L}}(x) = \frac{\sum e_i^4}{4\pi^2}\,x^2(1-x)\,. \qquad (1)$$

By Σe_i^4 we mean the sum of the fourth powers of the quark charges; the photon structure functions measure the sum of the fourth powers of the quark charges just as the total cross section for e^+e^- annihilation into hadrons measures the sum of the squares of the quark charges.

Note that according to (1), F_2 grows logarithmically as a function of Q^2 while F_{L} displays canonical Bjorken scaling. What we propose to calculate is the renormalization of the box diagram result (1) by the strong interactions. We will find that the Q^2 dependence suggested by the box diagram is correct, but the form of the structure functions is wrong.

2. The short-distance analysis

We wish to calculate the structure functions of the photon to lowest order in the electric charge but to all orders in the strong interactions.

We will first work with the moments of the structure functions. The short distance analysis expresses the moments of structure functions in terms of c-number coefficients multiplying matrix elements in the target state of the twist two operators. Explicitly

$$\int_0^1 \mathrm{d}x\, x^{n-2} F(x, q^2) = \sum_i C_i(q^2) \langle \gamma | O_i | \gamma \rangle . \tag{2}$$

Here $F(x, q^2)$ is any structure function; the O_i are operators of twist two and spin n, and the C_i are Wilson coefficients.

For a hadron target, the operators one must consider are the quark operators $\overline{\psi}\psi^n = \frac{1}{2}\overline{\psi}\gamma_{\mu_1} D_{\mu_2} ... D_{\mu_n}\psi$ and the gluon operators $GG^n = \frac{1}{2}G_{\mu_1\alpha} D_{\mu_2} ... D_{\mu_{n-1}} G^{\alpha}_{\mu_n}$ (G is the non-Abelian field strength tensor). The essential new feature of our problem is that there is an additional operator series that must be considered – the photon operators $FF^n = \frac{1}{2}F_{\mu_1\alpha} D_{\mu_2} ... D_{\mu_{n-1}} F^{\alpha}_{\mu_n}$ constructed from the electromagnetic field $F_{\alpha\beta}$ *.

Although the Wilson coefficient of FF^n is $O(\alpha)$, the matrix element in a photon state is $O(1)$. The quark and gluon operators, on the other hand, have Wilson coefficients of order one but matrix elements of order α. Thus, all are equally important.

Although the quark and gluon operators have unknown matrix elements in a photon state, the matrix element of the photon operator in a photon state is, to lowest order in α, simply equal to one – the free field theory value. This will be crucial in the analysis.

The Wilson coefficients satisfy a renormalization group equation

$$\left(\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial g} - \gamma\right) C(q^2/\mu^2, g, \alpha) = 0 . \tag{3}$$

Working to lowest order in α, we need not add a term $\beta'\, \mathrm{d}/\mathrm{d}e$, but in the anomalous dimension matrix γ, we will find it necessary to keep certain terms of order α. The solution of (3) is

$$C(q^2/\mu^2, g, \alpha) = T \exp \int_{\overline{g}}^{g} \mathrm{d}t \frac{\gamma(t)}{\beta(t)} C(1, \overline{g}(q^2/\mu^2, g), \alpha) , \tag{4}$$

where $\overline{g}$ is the usual effective coupling constant and T represents t-ordering.

We must now count powers of α in order to determine how accurately (4) must be evaluated. Let us write the column vector C as $C = \binom{\hat{C}}{D}$, where $\hat{C}$ are the coefficients of the quark and gluon operators and D is the coefficient of the photon operator. Then we must evaluate D to order α but $\hat{C}$ only to order one (D, of course, be-

* The factors of $\frac{1}{2}$ are included so that the spin averaged matrix element of a quark operator in a quark state or of a vector meson operator in a vector meson state is, in free field theory, equal to $p_{\mu_1} ... p_{\mu_n}$. With this definition we may use the anomalous dimensions and ratios of Wilson coefficients of Gross and Wilczek [1] and Zee, Wilczek, and Treiman [7].

gins at order α). It follows that if we define

$$T \exp \int_{\bar{g}}^{g} \mathrm{d}t \, \frac{\gamma(t)}{\beta(t)} = \left(\begin{array}{c|c} M & y \\ \hline x & z \end{array}\right),$$

then y may be set equal to zero, while M and z must be evaluated to order one, and x, which has no term of order one, must be evaluated to order α.

Writing γ as

$$\gamma = \left(\begin{array}{c|c} \hat{\gamma} & n \\ \hline k & q \end{array}\right)$$

we find now that n may be set equal to zero. Ostensibly q must be evaluated to order one, but q vanishes to this order, and so may be set equal to zero. $\hat{\gamma}$ must be computed to order one, and is therefore the usual anomalous dimension matrix computed in the absence of electromagnetism. And k must be considered to order α.

Once n and q are set equal to zero, (4) simplifies. We find

$$M(q^2/\mu^2, g, \alpha) = T \exp \int_{\bar{g}}^{g} \mathrm{d}g' \, \frac{\hat{\gamma}(g')}{\beta(g')},$$

$$x(q^2/\mu^2, g, \alpha) = \int_{\bar{g}}^{g} \mathrm{d}g' \, \frac{K(g', \alpha)}{\beta(g')} \, T \exp \int_{\bar{g}}^{g'} \mathrm{d}g'' \, \frac{\hat{\gamma}(g'')}{\beta(g'')},$$

$$z = 1 \,. \tag{5}$$

Our remaining task is to evaluate the integral for x. We will find that x and z give anomalous contributions that asymptotically dominate the normal scaling contribution from M.

Since k does not vanish when g' goes to zero, while β vanishes like g'^3, the integral defining x diverges as $\bar{g} \to 0$. Since we are interested in very large Q^2 (or very small $\bar{g}$), it is sufficient for us to evaluate the most divergent term. And since this divergence arises for very small g', we may use lowest order perturbation theory for k, β, and $\hat{\gamma}$.

Therefore, let us define

$$\beta = -\frac{bg^3}{8\pi^2} + \mathrm{O}(g^5) \,,$$

$$k = -\frac{re^2}{8\pi^2} + \mathrm{O}(e^2 g^2) \,,$$

$$\hat{\gamma} = \gamma_0 \, \frac{g^2}{8\pi^2} + \mathrm{O}(g^4) \,. \tag{6}$$

(Here b and r are positive; r is a row vector and γ_0 a matrix.) Moreover, we can expand γ_0 in terms of its eigenvalues λ_k and the corresponding projection matrices P_k, $\gamma_0 = \sum \lambda_k P_k$.

With these definitions, one may readily evaluate (5),

$$x(q^2/\mu^2, g, \alpha) = \frac{e^2 r}{2b\bar{g}^2} \sum_k \frac{P_k}{(1+\lambda_k/2b)} + \text{lower order}\,. \tag{7}$$

What is the relationship between this rather formal calculation and the box diagram? If one drops the factors $1/(1+\lambda_k/2b)$ one retrieves the free quark model result. In fact, since $\Sigma P_k = 1$ and $\bar{g}^2 \sim 8\pi^2/b \ln Q^2$, one would then obtain $x \sim e^2 r \ln Q^2/16\pi^2$ which (to leading logarithmic order) agrees with the box diagram, since the coefficient of the logarithmic growth of the box diagram is precisely $-\frac{1}{2}$ the mixing anomalous dimension, which in turn is $k = -re^2/8\pi^2$. The factors $1/(1+\lambda_k/2b)$ may therefore legitimately be regarded as the renormalization of the box diagram by the strong interactions.

3. A quantitative analysis

We first wish to verify that it is in fact the calculable contributions associated with x and z of eq. (5) that will dominate the photon structure functions at large Q^2.

We recall that the photon operator has a known matrix element, $\langle\gamma|FF^n|\gamma\rangle = 1$. The remaining twist-two operators, the quark and gluon operators with unknown matrix elements, we will denote as $\hat{O}_i$. Combining the formulas of the previous section, we find

$$\int_0^1 \mathrm{d}x\, x^{n-2} F(x, q^2) = \sum_i \left(M(q^2/\mu^2, g, \alpha)\, \hat{C}(1, \bar{g}^2, \alpha)\right)_i \langle\gamma|\hat{O}_i|\gamma\rangle$$
$$+ \sum_i \left(\frac{e^2 r}{2b\bar{g}^2} \sum_k \frac{P_k}{1+\lambda_k/2b}\right)_i \hat{C}_i(1, \bar{g}^2, \alpha) + D(1, \bar{g}^2, \alpha)\,. \tag{8}$$

What are the scaling properties of the various terms?

We consider first the transverse structure function F_2. For F_2, $\hat{C}$ and D approach constant limits as $Q^2 \to \infty$. Therefore, the third term in (8) is Q^2 independent for large Q^2. The second term, however, will grow like $\ln Q^2$, because of the factor $1/\bar{g}^2$. In the first term, M will scale with the usual anomalous dimensions for hadron targets (see eq. (5)), which means that the first term is asymptotically Q^2 independent for $n = 2$, and logarithmically suppressed for $n > 2$. Therefore, the dominant term is the logarithmically growing second term, and each moment of F_2 will grow like $\ln Q^2$.

As for the longitudinal structure function, $\hat{C}$ is now of order $\bar{g}^2$ while D is of order one, so that the second and third terms in (8) are now of equal importance. The first is still suppressed. Each moment of F_L will exhibit simple Bjorken scaling.

For a quantitative analysis, we will consider the theory with two quark triplets (p and p′) of charge $\frac{2}{3}$ and two triplets (n and λ) of charge $-\frac{1}{3}$.

In this theory there are three operators constructed from quark and gluon fields that we must consider

$$\hat{O} = \begin{bmatrix} \bar{\psi}\psi_S \\ GG \\ \bar{\psi}\psi_{NS} \end{bmatrix}$$

where $\bar{\psi}\psi_S$ is the SU(4) singlet quark operator and $\bar{\psi}\psi_{NS}$ is the non-singlet operator which transforms as $\bar{p}'p' + \bar{p}p + \bar{n}n + \bar{\lambda}\lambda$.

Our first task is to evaluate the matrix $\Sigma_k P_k/(1 + \lambda_k/2b)$ which appears in (7) and (8). The matrix γ_0 defined in (6) has the structure

$$\gamma_0 = \begin{bmatrix} s & -t & 0 \\ -u & v & 0 \\ 0 & 0 & s \end{bmatrix}, \tag{9}$$

where s, t, u, and v have been calculated [1] and are positive.

In terms of $d = 1 + (s + v)/2b + (sv - tu)/4b^2$, we find, after some algebra,

$$\sum \frac{P_k}{1 + \lambda_k/2b} = \begin{bmatrix} \frac{1}{d}\left(1 + \frac{v}{2b}\right) & \frac{t}{2bd} & 0 \\ \frac{u}{2bd} & \frac{1}{d}\left(1 + \frac{s}{2b}\right) & 0 \\ 0 & 0 & \frac{1}{1 + s/2b} \end{bmatrix}. \tag{10}$$

Moreover, for large Q^2 we may use the lowest order values

$$\hat{C} = e^2 \begin{bmatrix} \frac{5}{18} \\ 0 \\ \frac{1}{6} \end{bmatrix}, \tag{11}$$

$$r = \frac{e^2 u}{8\pi^2 T(R)} \left(\tfrac{10}{3} \quad 0 \quad 2\right). \tag{11}$$

Inserting (10) and (11) in (8), we obtain

$$\int_0^1 \mathrm{d}x\, x^{n-2} F_2(x, q^2) = \frac{e^4}{8\pi^2} \frac{u}{T(R)\, 2b\bar{g}^2} \frac{34}{27} \left[\frac{25}{34}\left(1 + \frac{v}{2b}\right)\frac{1}{d} + \frac{9}{34} \frac{1}{1 + s/2b}\right].$$

This differs from the free quark theory only by the factor in brackets, so we obtain finally

$$\lim_{Q^2\to\infty} \frac{\left(\int_0^1 \mathrm{d}x\, x^{n-2} F_2(x, q^2)\right)_{\text{exact}}}{\left(\int_0^1 \mathrm{d}x\, x^{n-2} F_2(x, q^2)\right)_{\substack{\text{box}\\ \text{diagram}}}} = \frac{25}{34}\left(1 + \frac{v}{2b}\right)\frac{1}{d} + \frac{9}{34}\frac{1}{1 + s/2b}\,. \tag{12}$$

For the longitudinal structure function, the calculation is only slightly more complicated.

The relevant Wilson coefficients have already been calculated [5]. They are

$$\hat{C} = \frac{e^2\bar{g}^2}{4\pi^2}\begin{bmatrix} \frac{5}{18} & C_2(R)/(n+1) \\ \frac{5}{18} & 4T(R)/(n+1)(n+2) \\ \frac{1}{6} & C_2(R)/(n+1) \end{bmatrix}$$

$$D = \frac{e^4}{4\pi^2}\,\frac{34}{27}\,\frac{4}{(n+1)(n+2)}\,. \tag{13}$$

Combining (13) with previous formulas, we now find

$$\lim_{Q^2\to\infty} \frac{\left(\int_0^1 \mathrm{d}x\, x^{n-2} F_\mathrm{L}(x, q^2)\right)_{\text{exact}}}{\left(\int_0^1 \mathrm{d}x\, x^{n-2} F_\mathrm{L}(x, q^2)\right)_{\substack{\text{box}\\ \text{diagram}}}} = 1 + \frac{9}{34}\left(1 + \frac{2}{n^2+n}\right)\frac{C_2(R)}{2b(1 + s/2b)}$$

$$+ \frac{25}{34}\left(1 + \frac{2}{n^2+n}\right)\frac{1}{2bd}\left[C_2(R)(1 + v/2b) + \frac{4T(R)}{n+2}\,\frac{t}{2b}\right]. \tag{14}$$

Eqs. (12) and (14) are our main results.

Qualitatively, we find that the zeroth and higher moments of F_2 are suppressed relative to the free quark model while moments of F_L are enhanced.

For example, from (12) we find that $\int_0^1 \mathrm{d}x\, F_2(x)$ is suppressed by a factor 0.741 relative to the free quark model, while $\int_0^1 \mathrm{d}x\, x^2 F_2(x)$ is suppressed by a factor 0.546. On the other hand, from (14), $\int_0^1 \mathrm{d}x\, F_\mathrm{L}(x)$ is enhanced by a factor 1.216 and $\int_0^1 \mathrm{d}x\, x^2 F_\mathrm{L}(x)$ is enhanced by a factor 1.099.

More interesting than the moments are the structure functions themselves. These can be obtained by taking an inverse Mellin transform. In fact, if the moments $a(n)$ of the function $F(x)$ are defined by $a(n) = \int_0^1 \mathrm{d}x\, x^{n-2} F(x)$, then $F(x)$ can be retrieved from

$$F(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \mathrm{d}n\, x^{-n+1}\, a(n)\,, \tag{15}$$

where the integration contour runs to the right of all singularities of $a(n)$ in the complex plane. We have carried out the integration (15) numerically, with results that are shown in figs. (3) and (4). For $a(n)$ we use, of course, the box diagram free quark moments times the correction factors (12) and (14).

It will be seen from figs. 3 and 4 that F_2 is renormalized by the strong interactions almost beyond recognition, while for F_L, the free quark model is a fairly good description except at very small x.

The functions graphed in figs. 3 and 4 we will refer to as the scaling structure functions of the photon. Their precise relation to possible future experiments is the following. At any given value of x (or for any given moment) the experimental values should, at large enough Q^2, agree with the scaling structure functions. However, the approach to the scaling structure functions may be non-uniform as a function of x, or of the moment number n, and at any given Q^2, the actual structure functions may differ from the scaling ones for large enough or small enough x. Indeed, we expect such non-uniformity, because of the fact that anomalous dimensions become large, and have large corrections, both for large and for small n [6].

Although the integral (15) must be performed numerically, some qualitative features can easily be determined analytically. The behavior as $x \to 0$ is determined by the rightmost singularity of $a(n)$ in the complex n plane. For both F_2 and F_L the rightmost singularity is a pole at approximately $n = 1.5964$, and correspondingly the scaling forms of both F_2 and F_L diverge for small x as $x^{-0.5964}$. The coefficient of

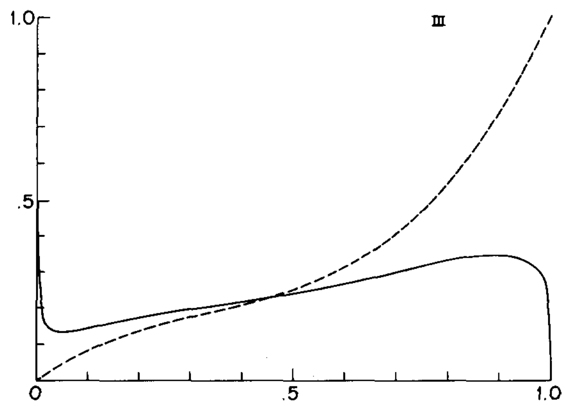


Fig. 3. The structure function F_2 of the photon in the free quark theory (dashed line) and in the interacting quark-gluon theory (solid line). F_2 is given in units of $\Sigma e_i^4 \ln Q^2/16\pi^2$.

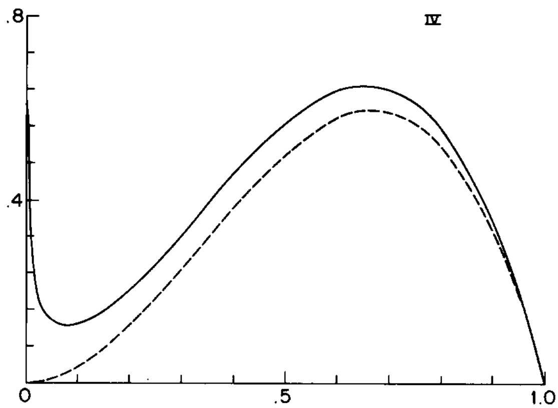


Fig. 4. The longitudinal structure function of the photon in the free quark theory (dashed line) and in the interacting theory (solid line), in units of $\Sigma e_i^4/16\pi^2$.

this singular term is rather small; in the units of the diagram, it is roughly 0.012 for F_2.

The behavior of the scaling structure functions as $x \to 1$ is, on the other hand, determined by the behavior of $a(n)$ for large n. For F_2 we find, in the free quark model, $a(n) \sim 1/n$, which means $F_2(x)$ does not vanish as $x \to 1$. In fact the free quark $F_2(x)$ has its maximum at $x = 1$. But the exact moments of $F_2(x)$ vanish for large n as $1/(n \ln n)$, which means that for $x \to 1, F_2(x)$ will vanish as $|\ln(1-x)|^{-1/2}$, which is to say, very slowly indeed. For F_L, on the other hand, we find that the free quark result is exact for large n, and hence, that it becomes exact for $x \to 1$.

We emphasize again that the approach to asymptotia will be non-uniform as a function of x, and in particular we certainly do not expect either F_2 of F_L to diverge for $x \to 0$ at any given value of Q^2.

4. The corrections

We must now ask how large the corrections to our asymptotic theorems are and to what extent we expect that it will be possible to test the theory at moderate energies. We see several issues.

(a) First, we should stress that our calculation will be relevant at reasonable values of Q^2 only if the strong interactions actually are a strongly coupled field theory for momenta of the order of the quark masses. In fact, if one reexamines carefully the steps that lead from the integral for x in (5) to the asymptotic form (7), one will see that the validity of (7) requires not just $\bar{g}^2 \ll 1$ but also $\bar{g}^2 \ll g^2$. If the physical coupling is of order one, the latter condition is not serious, but if the physical coupling is small, the latter condition requires $Q^2 \gg m^2 \exp(1/bg^2)$. In hoping that our formulas are relevant to experiment, we are thus counting on the "standard" idea that the strong interactions are strong at energies of a few hundred MeV, and rapidly become weak due to renormalization effects.

(b) To what extent are the corrections to our asymptotic theorems calculable? We have so far only evaluated the leading divergence as $\bar{g} \to 0$ of the crucial integral for x in (5). A more careful study of that integral shows that the exact answer has the following structure.

For $n > 2$, so that the anomalous dimensions of the quark and gluon operators are positive, the moments of F_2 behave as follows:

$$\int_0^1 \mathrm{d}x\, x^{n-2} F_2(x) = \frac{A_n}{\bar{g}^2} + B_n + \mathrm{O}(\bar{g}^2) + \sum C_i^n \bar{g}^{\lambda_i^n/b} (1 + \mathrm{O}(\bar{g}^2)) . \qquad (16)$$

Here λ_i^n are the eigenvalues of the spin n anomalous dimension matrix. The numbers A_n we have already calculated (eq. (12)). For B_n one can give an explicit formula in terms of one-loop Wilson coefficients and the two-loop contributions to β and γ. β has been calculated to two-loop order [7], but unfortunately the two-loop contribution to γ is not known. By comparison, the C_i^n depend on the Wilson coefficients and matrix elements of the quark and gluon operators, and certainly cannot be calculated with present methods.

For $n = 2$ the integral in (5) contains a term $\int_{\bar{g}}^{g} (\mathrm{d}g'/g')$ that diverges logarithmically as $\bar{g} \to 0$, leading to a structure

$$\int_0^1 \mathrm{d}x\, F_2(x) = \frac{A}{\bar{g}^2} + B \ln \bar{g} + C_0 + \sum C_i \bar{g}^{\lambda_i/2b} (1 + \mathrm{O}(\bar{g}^2)) . \qquad (17)$$

Here $A/\bar{g}^2$, as we have noted, is 0.741 times the box diagram moment, and B could be calculated if the two-loop anomalous dimensions were known, but C cannot be determined from perturbation theory.

For F_L the results are similar, except that the right-hand side of (16) and (17) would be multiplied by a factor $\bar{g}^2$, and except that the determination of B would require a knowledge of certain Wilson coefficients to two-loop order.

To determine how large Q^2 must be to test the sum rules, it would be quite valuable to evaluate the leading corrections (proportional to B) to the sum rules, and especially to see whether the sign of B is such as to increase or diminish the difference between the exact moments and the free quark model results.

(c) In this paper we have ignored all quark masses, as is appropriate at asymptotically large Q^2. Suppose, however, that the photon structure functions are actually measured at, say, $Q^2 = 5\ \mathrm{GeV}^2$. At such Q^2, a short distance analysis may possibly be relevant, and it would probably be quite safe to ignore the p, n, and λ masses. But it would be a poor approximation to ignore the charmed quark mass. It would be necessary, and perfectly feasible, to use the heavy quark expansion that has been developed previously [8], together with the method of this paper, to compute the photon structure functions.

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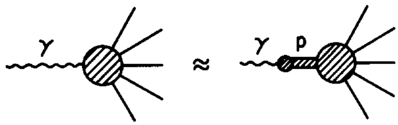

Fig. 5. Vector meson dominance.

(d) Finally, we would like to estimate the importance of the correction terms proportional to C in (16) and (17).

There is no way, using existing methods, to evaluate C, so the only rigorous way to test the sum rules is to consider Q^2 so large that the C terms are unimportant.

The best that we can do is to use vector meson dominance to guess as to the magnitude of C.

Vector meson dominance asserts that the coupling (fig. 5) of a soft isovector photon is $(e/f_\rho)\,[-m_\rho^2/(p^2 - m_\rho^2)]$ times the coupling of a ρ meson, with a similar form for isoscalar photons. The factor in brackets is the ρ propagator; we will take-on shell target photons, $p^2 = 0$. The most optimistic application of vector meson dominance would involve applying vector meson dominance to each of the target photons, and leads to the conclusion (fig. 6) that $F^\gamma = e^2/f_\rho^2\, F^\rho$ (where F is any structure function).

This relation is certainly wrong, because F^ρ will exhibit normal scaling governed by quark-gluon operators, while F^γ has additional, anomalous contributions. In fact, the entire structure function F^ρ will scale like the C terms of (16) and (17); the A and B terms have no analogue in F^ρ.

Thus, vector meson dominance cannot be used to estimate the entire photon structure function, but perhaps it can be used to estimate the C terms, which are present both for the photon and for the ρ meson.

For the ρ structure functions we have, of course, no experimental information, but we can use the energy-momentum tensor sum rule to get some idea. In fact, at large Q^2 we expect [1]

$$\int_0^1 \mathrm{d}x\, F_2^\rho(x) = \langle e_i^2\rangle \frac{T(R)}{2C_2(R) + T(R)}\,,$$

where $\langle e_i^2\rangle$ is the average squared quark charge. In the four quark triplet theory, $\langle e_i^2\rangle = \frac{5}{18}e^2$, and $T(R)/(2C_2(R) + T(R)) = \frac{3}{7}$, so $\int_0^1 \mathrm{d}x\, F_2(x) = \frac{15}{126}e^2$. Such an estimate

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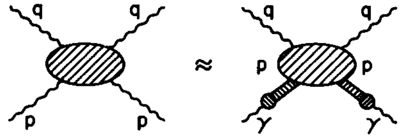

Fig. 6. The most optimistic (and incorrect) application of vector meson dominance.

is fairly accurate for the proton at $Q^2 = 5$ GeV^2. This leads to

$$\Big(\int_0^1 \mathrm{d}x\, F_2^\gamma(x)\Big)_{\substack{\rho\ \text{meson}\\ \text{contribution}}} = \frac{15}{126}\frac{e^4}{f_\rho^2}\ .$$

We may roughly allow for the presence of isoscalar vector mesons (ω, ϕ, ψ) by multiplying by 2, so we estimate

$$\Big(\int_0^1 \mathrm{d}x\, F_2^\gamma(x)\Big)_{\substack{\text{vector meson}\\ \text{part}}} = \frac{15}{63}\frac{e^4}{f_\rho^2}\ . \tag{18}$$

By comparison, the leading (A) term in (14), which we have explicitly evaluated, gives

$$0.741\,\Big(\sum e_i^4\Big)\frac{1}{25\pi^2\bar{\alpha}}\ ,$$

where 0.741 is the correction factor that we have evaluated, Σe_i^4 is the sum of the fourth powers of the quark charges, and $1/25\pi^2\bar{\alpha} \sim \ln Q^2/12\pi^2$ comes from the box diagram *. In the four quark triplet model we thus have

$$\Big(\int_0^1 \mathrm{d}x\, F_2^\gamma(x)\Big)_{\substack{\text{leading}\\ \text{term}}} = (0.741)\left(\frac{34}{27}\right)\frac{e^4}{25\pi^2\bar{\alpha}}\ . \tag{19}$$

Using $f_\rho^2/4\pi^2 \approx 2.56$, we now find that the ratio of (18) to (19) is about $1.9\,\bar{\alpha}$. Therefore, our asymptotic theorem will be valid when $1.9\,\bar{\alpha}$ is small.

At Q^2 of, say, 5 GeV^2, this correction term of order $1.9\,\bar{\alpha}$ is probably fairly small compared to the leading term, but large enough as to obscure the factor of 0.741 that distinguishes the interacting field theory from the free quark model.

A similar estimate, which we will not reproduce, suggests that the vector meson dominance contribution to $\int_0^1 \mathrm{d}x\, F_\mathrm{L}(x)$ is of order $0.9\,\bar{\alpha}$ relative to the leading term.

Qualitatively, we expect the vector dominance contribution to be less important for higher moments, or large x, since the ρ, as a bound state, probably has structure functions which vanish more rapidly for large x than those of the photon.

Since we not know the x dependence of the ρ structure functions, we cannot make very reliable statements. However, to get some idea we will use for the ρ the guess of Farrar [9] for the π structure functions. Using Farrar's meson structure functions, we find that at $x = 0.6$, the vector dominance contribution to F_2 is fractionally about

* Following Gross and Wilczek [1], by $\bar{\alpha}$ we mean $\bar{g}^2/4\pi^2$. This differs by a factor $1/\pi$ from the usual definition in quantum electrodynamics.

$1.3\,\bar{\alpha}$; at $x = 0.8$, it is about $\frac{1}{2}\bar{\alpha}$; and at $x = 0.9$ it is about $0.3\,\bar{\alpha}$. While these numbers are certainly not reliable, they suggest that for x of, say, 0.8, the uncalculable vector dominance corrections to the photon structure functions may be small even for moderate Q^2.

5. Some additional questions

We have considered only the two structure functions, F_2 and F_L, that determine the cross section for unpolarized targets. Given polarized photon targets, there are two additional structure functions; it is clear that these could be analyzed as we have analyzed F_2 and F_L. Using the notation of Ahmed and Ross [4], W_3, which scales in the free quark model, will be analogous to F_L, and W_4, which has a logarithmic growth in the free quark model, will be analogous to F_2.

One may also consider the annihilation structure functions for the process $e^+e^- \to \gamma$ + hadrons. Mueller [10] and Coote [11] have shown that if the strong interactions were described by a ϕ^4 or Yukawa interaction, then the reaction $e^+e^- \to A$ + hadrons (where A is one observed particle) would have a renormalization group structure analogous to that of electroproduction. If such results can be established in non-Abelian gauge theories, we expect that there will be anomalous theorems for $e^+e^- \to \gamma$ + hadrons similar to those we have derived here for deep inelastic scattering. (For such results to make theoretical and experimental sense, it would be necessary to somehow subtract the large production of photons from π^0 decays.)

6. Conclusion

Even a crude measurement of the photon structure functions would distinguish a fractionally charged quark model from integrally charged quark models. The sum of the fourth powers of the quark charges is simply much smaller if the quarks have fractional charges. The corrections we have found are too subtle to be important at the level of distinguishing fractional from integral quark charges.

But theoretically the most extraordinary feature of this reaction is that the answer is calculable, with no undetermined constants, yet the free quark answer is *wrong*. In this respect the status of the photon structure functions is practically unique.

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