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CP Violation*

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Abstract

We review the phenomenology of CP violation. The first part is a general discussion of CP violation in meson decays and in fermion electric dipole moments. The second part describes CP violation in the Standard Model. The third part describes CP violation beyond the Standard Model. Our discussion is free of phase conventions and uses one language for the K and B systems, which gives further insight into the advantages of measuring CP violation in the B system.

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Introduction

One of the most intriguing aspects of high energy physics is CP violation. On the experimental side, it is one of the least tested aspects of the Standard Model. There is only one CP violating parameter that has been unambiguously measured, that is the ϵ parameter in the neutral K system [1]. A genuine testing of the Kobayashi–Maskawa picture of CP violation [2] in the Standard Model [3 – 5] awaits the building of B factories that would provide a second, independent, measurement of CP violation [6]. On the theoretical side, the Standard Model picture of CP violation has two major difficulties. First, CP violation is necessary for baryogenesis [7], but the Standard Model CP violating processes seem unable to produce the observed baryon asymmetry of the universe. Second, an extreme fine tuning is needed in the CP violating part of the QCD lagrangian in order that its contribution to the electric dipole moment of the neutron [8, 9] does not exceed the experimental upper bound [10, 11]. This suggests that an extension of the Standard Model, such as the Peccei-Quinn symmetry [12], is required.

In this series of lectures we concentrate on three classes of CP violating processes where the Standard Model will be tested and the existence of new physics may be revealed: neutral K decays into two pions, neutral B decays into final CP eigenstates, and fermionic electric dipole moments. The best determination of the CP violating parameters in the Standard Model will come from the neutral meson decays and we put our emphasis on these.

The first part of these lectures is a general discussion of CP violation in meson decays. We define three types of CP violation in neutral meson systems: CP violation in decay, CP violation in mixing, and CP violation in the interference of mixing and decay. We describe how each of the three types can be observed and we explain the difficulties in the respective theoretical calculations. We analyze the differences between the K and the B systems in both experiment and theory. *The whole discussion is free of phase conventions and uses one language for both K and B mesons.*

The second part of the lectures describes the CKM picture of CP violation within the Standard Model. We use unitarity triangles to explain the features of CP violation in K, B and B_s decays. We accompany this with a detailed calculation, updated with recent experimental measurements and theoretical considerations (such as Heavy Quark Symmetry). The predictions for CP asymmetries in neutral B decays are presented in a novel way which makes comparison to models of new physics more straightforward.

The third part of these lectures is devoted to theories beyond the Standard Model. We analyze in detail CP violation in several extensions of the Standard Model: An extension of the quark sector with an $SU(2)_L$ down-like singlet; Extensions the Higgs sector which maintain Natural Flavor Conservation; Extensions of the gauge sector into Left-Right Symmetry which allow CP to be only spontaneously broken; and Supersymmetry. For each of these models we analyse the constraints and predictions concerning CP violation. We end this part by presenting the predictions of various schemes for quark mass matrices for CP asymmetries in B decays.

In preparing this series of lectures, I have used the following reviews: Ref. [13] for a general review; Ref. [14] for the K system; Refs. [15 – 18] for the B system; Refs. [19 – 21] for electric dipole moments; Refs. [22 – 24] for the CKM picture. In these reviews the reader may find more complete lists of references: I here included only those references which have actually been used in preparing these lectures.

I. CP VIOLATION IN NEUTRAL MESON SYSTEMS

1. Formalism and Notations

1.1. CP -CONJUGATE DECAYS

We are interested in pairs of decay processes that are related by a CP transformation. If P and $\bar{P}$ are CP conjugate mesons and f and $\bar{f}$ are CP conjugate states, then we denote by A and $\bar{A}$ the the two CP conjugate decay amplitudes:

$$A \equiv \langle f | H | P \rangle , \quad \bar{A} \equiv \langle \bar{f} | H | \bar{P} \rangle . \tag{1.1}$$

There are two types of phases that may appear in A and $\bar{A}$. *Weak phases* are parameters in the Lagrangian which violate CP . They appear in A and $\bar{A}$ with opposite signs. They usually appear in the electroweak sector of the theory and hence the name "weak". *Strong phases* appear in scattering or decay amplitudes even when the Lagrangian is real. They do not violate CP and appear in A and $\bar{A}$ with the same sign. Their origin is in the possible contribution from intermediate on-shell states in the decay process, namely in the absorptive part of an amplitude that has contributions from coupled channels. Usually the relevant rescattering is due to strong interactions and hence the name "strong".

It is useful to factorize A into three: the absolute value of A; a strong phase shift δ which is the result of final state interaction (and is CP invariant); and a weak phase ϕ which is CP violating. Then, if several amplitudes contribute to $P^0 \to f$,

$$A = \sum_i A_i e^{i\delta_i} e^{i\phi_i}; \quad \bar{A} = e^{-2i\xi_P} e^{+2i\xi_f} \sum_i A_i e^{i\delta_i} e^{-i\phi_i}, \tag{1.2}$$

where A_i are real, ξ_P and ξ_f are phases related to the CP transformation law for P and f, respectively (see below). If f is a CP eigenstate then $e^{-2i\xi_f} = \pm 1$, according to whether f is CP even or odd. The notation $a_i \equiv A_i e^{i\phi_i}$ is also common in literature.

1.2. MIXING OF NEUTRAL MESONS

We consider a neutral meson P^0 and its antiparticle $\bar{P}^0$ [25]. An arbitrary neutral P-meson state

$$a\left|P^0\right\rangle + b\left|\bar{P}^0\right\rangle \tag{1.3}$$

is governed by the time-dependent Schrödinger equation

$$i\frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix} = H\begin{pmatrix} a \\ b \end{pmatrix} \equiv (M - \tfrac{i}{2}\Gamma)\begin{pmatrix} a \\ b \end{pmatrix}. \tag{1.4}$$

Here M and Γ are 2×2 hermitian matrices. CPT invariance guarantees $H_{11} = H_{22}$. In H, the anti-hermitian part – $i\Gamma$ – describes the exponential decay of the P-meson system, while the hermitian part – M – is called a mass matrix. The non-diagonal terms would be important in the discussion of CP violation:

$$\begin{aligned} M_{12} &= \left\langle P^0\right| H_{\Delta p=2}\left|\bar{P}^0\right\rangle + \mathrm{P}\sum_n \frac{\left\langle P^0\right| H_{\Delta p=1}\left|n\right\rangle \left\langle n\right| H_{\Delta p=1}\left|\bar{P}^0\right\rangle}{m_P - E_n}, \\ \Gamma_{12} &= 2\pi\sum_n \rho_n \left\langle P^0\right| H_{\Delta p=1}\left|n\right\rangle \left\langle n\right| H_{\Delta p=1}\left|\bar{P}^0\right\rangle, \end{aligned} \tag{1.5}$$

where P stands for principal value and ρ_n is the density of the state n.

The mass eigenstates are

$$\begin{aligned} \left|P_1\right\rangle &= p\left|P^0\right\rangle + q\left|\bar{P}^0\right\rangle, \\ \left|P_2\right\rangle &= p\left|P^0\right\rangle - q\left|\bar{P}^0\right\rangle, \end{aligned} \tag{1.6}$$

with the normalization condition

$$|q|^2 + |p|^2 = 1. \tag{1.7}$$

You may be puzzled by the form of (1.6). First, P_1 and P_2 are not necessarily orthogonal states:

$$\langle P_1|P_2\rangle = |p|^2 - |q|^2. \tag{1.8}$$

If $\Gamma_{12} = 0$ then H would be the sum of a unit matrix (times a complex number) and a hermitian matrix and its eigenvectors would be orthogonal. In the usual treatment of field theory, one indeed diagonalizes M and treats Γ as interaction among the orthogonal states. Here we incorporate Γ into our effective hamiltonian which has, therefore, non-orthogonal eigenvectors. In other words, P_1 and P_2 are resonances and not elementary particles. Furthermore, if $\Gamma_{12} \neq 0$ but $\arg(\Gamma_{12}/M_{12}) = 0$, then P_1 and P_2 would still be orthogonal, in the sense that (1.8) would vanish. This case corresponds to P_1 and P_2 carrying different quantum numbers under a good symmetry (CP). Second, there are no four independent components p_i and q_i in (1.6). The relations $p_1 = p_2$, $q_1 = -q_2$ are a result of $H_{11} = H_{22}$, namely of CPT.

The eigenvalues of H are

$$\mu_{1,2} = M_{1,2} - \tfrac{i}{2}\Gamma_{1,2}, \tag{1.9}$$

where M_i and Γ_i are the mass and the decay width, respectively, of P_i. We further define

$$\Delta\mu \equiv \mu_2 - \mu_1 \equiv \Delta M - \tfrac{i}{2}\Delta\Gamma. \tag{1.10}$$

The eigenvalue problem,

$$\det\left(M - \tfrac{i}{2}\Gamma - \mu\mathbf{1}\right) = 0, \tag{1.11}$$

leads to the condition

$$(\Delta\mu)^2 = 4(M_{12} - \tfrac{i}{2}\Gamma_{12})(M_{12}^* - \tfrac{i}{2}\Gamma_{12}^*) \tag{1.12}$$

or, equivalently,

$$\begin{aligned}(\Delta M)^2 - \tfrac{1}{4}(\Delta\Gamma)^2 =& 4(|M_{12}|^2 - \tfrac{1}{4}|\Gamma_{12}|^2),\\ \Delta M \Delta\Gamma =& 4\mathrm{Re}(M_{12}\Gamma_{12}^*).\end{aligned} \tag{1.13}$$

For the ratio q/p we find

$$\frac{q}{p} = \frac{-\Delta\mu}{2(M_{12} - \frac{i}{2}\Gamma_{12})} = -\frac{2(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)}{\Delta\mu}. \tag{1.14}$$

Of p and q only the ratio q/p has physical significance. First, there is the normalization condition (1.7). Second, $\arg(q/p^*)$ is just an overall common phase for $|P_1\rangle$ and $|P_2\rangle$.

1.3. PHASE CONVENTIONS

There is some freedom in defining phases which has to be clarified. (We follow here the discussion in ref. [13].) In particular, each time we define a *CP* violating observable, we would like to verify that it is independent of phase conventions. The states P^0 and $\bar{P}^0$ are related through *CP* transformation:

$$CP\left|P^0\right\rangle = e^{2i\xi}\left|\bar{P}^0\right\rangle, \quad CP\left|\bar{P}^0\right\rangle = e^{-2i\xi}\left|P^0\right\rangle, \tag{1.15}$$

where ξ is an *arbitrary* phase. The freedom in defining phases is related to the fact that P^0 and $\bar{P}^0$ are defined by strong interactions which conserve flavor. Therefore, a phase transformation,

$$\left|P_\zeta^0\right\rangle = e^{-i\zeta}\left|P^0\right\rangle, \quad \left|\bar{P}_\zeta^0\right\rangle = e^{+i\zeta}\left|\bar{P}^0\right\rangle, \tag{1.16}$$

has no physical effects. This invariance is just the Strangeness, Charm or Beauty symmetry of strong interactions for K, D or B, respectively. In the new basis,

CP transformations take the form

$$(CP)_\zeta \left| P^0_\zeta \right\rangle = e^{2i(\xi-\zeta)} \left| \bar{P}^0_\zeta \right\rangle, \quad (CP)_\zeta \left| \bar{P}^0_\zeta \right\rangle = e^{-2i(\xi-\zeta)} \left| P^0_\zeta \right\rangle. \tag{1.17}$$

The various quantities discussed in this chapter transform according to

$$\begin{aligned} M^\zeta_{12} =& e^{2i\zeta} M_{12}, \quad \Gamma^\zeta_{12} = e^{2i\zeta}\Gamma_{12}, \quad (q/p)_\zeta = e^{-2i\zeta}(q/p), \\ A_\zeta =& e^{-i\zeta} A, \quad \bar{A}_\zeta = e^{i\zeta}\bar{A}. \end{aligned} \tag{1.18}$$

Furthermore, from the transformation of states (1.16), and the transformation of q/p in (1.18), we find that

$$\left| P_{1\zeta} \right\rangle = e^{i\zeta'} \left| P_1 \right\rangle, \quad \left| P_{2\zeta} \right\rangle = e^{i\zeta'} \left| P_2 \right\rangle, \tag{1.19}$$

namely both mass eigenstates are rotated by a common phase factor, which has no physical significance.

An alternative common notation is to define $\bar{\epsilon}$ such that

$$p = \frac{1+\bar{\epsilon}}{\sqrt{2(1+|\bar{\epsilon}|^2)}}, \quad q = \frac{1-\bar{\epsilon}}{\sqrt{2(1+|\bar{\epsilon}|^2)}}. \tag{1.20}$$

Note that the normalization condition (1.7) is explicitly incorporated and, furthermore, part of the freedom in phases is used to set $\mathrm{Im}(q) = -\mathrm{Im}(p)$.

2. The Three Types of *CP* Violation in Meson Decays

We distinguish between three types of *CP* violation:

(*i*) *CP* violation in decay.

The following quantity is independent of phase conventions and physically meaningful:

$$\left|\frac{\bar{A}}{A}\right| = \left|\frac{\sum_i A_i e^{i\delta_i} e^{-i\phi_i}}{\sum_i A_i e^{i\delta_i} e^{i\phi_i}}\right|. \tag{2.1}$$

When *CP* is conserved, the weak phases ϕ_i are all equal. Therefore, eq. (2.1) implies

$$|\bar{A}/A| \neq 1 \quad \Longrightarrow \quad CP \text{ violation.} \tag{2.2}$$

We call this type of *CP* violation CP *violation in decay* or *direct* CP *violation.* It results from the interference among various decay amplitudes that lead to the same final state. *CP* asymmetries in charged meson decays are of this type.

(*ii*) *CP* violation in mixing.

The following quantity is independent of phase conventions and physically meaningful:

$$\left|\frac{q}{p}\right|^2 = \left|\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\right|. \tag{2.3}$$

When *CP* is conserved, the relative phase between M_{12} and Γ_{12} vanishes. Therefore, eq. (2.3) implies

$$|q/p| \neq 1 \quad \Longrightarrow \quad CP \text{ violation.} \tag{2.4}$$

We call this type of *CP* violation CP *violation in mixing* or *indirect* CP *violation.* It results from the mass eigenstates being different from the *CP* eigenstates.

CP asymmetries in semileptonic decays are of this type. In the notation (1.20) we have

$$|q/p| = |(1-\bar{\epsilon})/(1+\bar{\epsilon})|, \tag{2.5}$$

so that CP violation in mixing is related to $\text{Re}(\bar{\epsilon}) \neq 0$.

(iii) CP violation in the interference of mixing and decay.

We denote by $A_{f_{CP}}$ the amplitude for P^0 decay into a final CP eigenstate f_{CP}. Then the following quantity is independent of phase conventions and physically meaningful:

$$\lambda \equiv \frac{q}{p}\frac{\bar{A}_{f_{CP}}}{A_{f_{CP}}}. \tag{2.6}$$

When CP is conserved $|q/p| = 1$, $|\bar{A}_{f_{CP}}/A_{f_{CP}}| = 1$ and the relative phase between (q/p) and $(\bar{A}_{f_{CP}}/A_{f_{CP}})$ vanishes. Therefore, eq. (2.6) implies

$$\lambda \neq 1 \quad \Longrightarrow \quad CP \text{ violation}. \tag{2.7}$$

CP asymmetries in neutral meson decays into CP eigenstates are of this type.

There are several important points concerning (2.7):

a. CP violation in decay (2.2) is sufficient for (2.7) through $|\lambda| \neq 1$.

b. CP violation in mixing (2.4) is sufficient for (2.7) through $|\lambda| \neq 1$.

c. Neither (2.2) nor (2.4) is *necessary* for (2.7) to realize. In fact, the theoretically favorable situation is when $|q/p| = 1$ and $|\bar{A}/A| = 1$, yet $\text{Im}\lambda \neq 1$, namely λ is a pure phase. The point is that in this case there are no hadronic uncertainties in the calculation of λ, as will be discussed in chapter 5. We will call CP violation of the form

$$|\lambda| = 1, \quad \text{Im}\lambda \neq 0, \tag{2.8}$$

CP violation in the interference of mixing and decay.

d. Take the decay amplitudes of P^0 into two different final CP eigenstates, A_a and A_b. A nonvanishing difference between λ_a and λ_b,

$$\lambda_a - \lambda_b = \frac{q}{p}\left(\frac{\bar{A}_a}{A_a} - \frac{\bar{A}_b}{A_b}\right) \neq 0, \tag{2.9}$$

would establish the existence of CP violation in $\Delta P = 1$ processes. Yet, unlike the case of direct CP violation, no nontrivial strong phases are necessary.

3. K and B Mesons

Discussing CP violation for the most general neutral meson system is extremely complicated and not very illuminating. Therefore, we will concentrate in two specific types of neutral meson systems: the case of "small phases" and the case of "small lifetime difference". In the end, there are three neutral meson systems useful for our understanding of CP violation, and they correspond to the two classes: in the neutral K system all relevant phases are small, while in the neutral B and B_s systems the two mass eigenstates have similar lifetimes. (In the D system the effects are small and arise mainly from long distance physics. Top quarks are likely to decay before T mesons form.) Thus, in this chapter we describe the K and the B systems.

3.1. THE NEUTRAL K SYSTEM

The two neutral K meson states differ significantly in their lifetimes [26]:

$$\tau_S = (0.8922 \pm 0.0020) \times 10^{-10}\ \text{s}, \quad \tau_L = (5.17 \pm 0.04) \times 10^{-8}\ \text{s}, \tag{3.1}$$

where the sub-indices S and L stand for Short and Long, respectively. We choose

$$\begin{aligned} |K_1\rangle &= |K_S\rangle\,, \quad |K_2\rangle = |K_L\rangle\,, \\ \Delta\Gamma &\equiv \Gamma_L - \Gamma_S < 0. \end{aligned} \tag{3.2}$$

The amplitudes of the states K_S and K_L at time t can be written as

$$a_S(t) = a_S(0)e^{-iM_S t}e^{-\frac{1}{2}\Gamma_S t}, \quad a_L(t) = a_L(0)e^{-iM_L t}e^{-\frac{1}{2}\Gamma_L t}. \tag{3.3}$$

The mass difference between the two neutral kaons is measured to be

$$\Delta M \equiv M_L - M_S = (3.522 \pm 0.016) \times 10^{-15}\ GeV. \tag{3.4}$$

Eqs. (3.1) and (3.4) together imply a useful approximate relation,

$$\Delta\Gamma_K \approx -2\Delta M_K. \tag{3.5}$$

Next, we turn to the calculation of

$$\frac{q}{p} = -\frac{2(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)}{\Delta M - \frac{i}{2}\Delta\Gamma}. \tag{3.6}$$

We define a phase ϕ_{12} according to

$$\frac{M_{12}}{\Gamma_{12}} = -\left|\frac{M_{12}}{\Gamma_{12}}\right| e^{i\phi_{12}}. \tag{3.7}$$

As CP violating effects in the K system are known to be small, we have $\phi_{12} \ll 1$. Solving (1.13) to first order in ϕ_{12} gives

$$\Delta M = 2|M_{12}|, \quad \Delta\Gamma = -2|\Gamma_{12}|. \tag{3.8}$$

Consequently, to first order in ϕ_{12}, (3.7) is equivalent to

$$\frac{M_{12}}{\Gamma_{12}} = \frac{\Delta M}{\Delta\Gamma}(1 + i\phi_{12}). \tag{3.9}$$

In any given phase convention

$$\Gamma_{12} = |\Gamma_{12}|e^{-2i\xi}. \tag{3.10}$$

Using (3.9) and (3.10), we get from (3.6):

$$\frac{q}{p} = e^{2i\xi}\left[1 - i\phi_{12}\frac{1 + i\frac{\Delta\Gamma}{2\Delta M}}{1 + \left(\frac{\Delta\Gamma}{2\Delta M}\right)^2}\right]. \tag{3.11}$$

Note that to a good approximation q/p is a pure phase. Actually (3.11) implies that the CP transformation law is $CP\left|K^0\right\rangle = e^{2i\xi}\left|\bar{K}^0\right\rangle$. Indeed we experimentally know that the K_S and K_L states are to a good approximation CP eigenstates. The violation of this approximation is of order $\phi_{12} = \mathcal{O}(10^{-3})$. In the calculation of the deviation from $|q/p| = 1$ there are significant hadronic uncertainties. They will be discussed in detail later. Here we just mention that they arise from a parameter called B_K which introduces an overall uncertainty of a factor of 2-3 in $|q/p| - 1$.

3.2. THE NEUTRAL B SYSTEM

The two neutral B mesons are expected to have a negligible difference in lifetimes,

$$\Delta\Gamma/\Gamma = \mathcal{O}(10^{-2}). \tag{3.12}$$

(Note that $\Delta\Gamma$ has *not* been experimentally measured. (3.12) is a theoretical statement based on experimental evidence, as discussed below.) We choose to define

$$\begin{aligned} |B_1\rangle =& |B_L\rangle\,, \quad |B_2\rangle = |B_H\rangle\,, \\ \Delta M \equiv& M_H - M_L > 0, \end{aligned} \tag{3.13}$$

where the sub-indices L and H stand for Light and Heavy. Note that (1.12) and (1.14) now lead to

$$\begin{aligned} \Delta M =& 2|M_{12}|, \quad \Delta\Gamma = 2\mathrm{Re}(M_{12}\Gamma_{12}^*)/|M_{12}|, \\ q/p =& -|M_{12}|/M_{12}. \end{aligned} \tag{3.14}$$

The time evolution of $\left|B^0_{phys}\right\rangle$, an initially pure B^0 ($a_L(0) = a_H(0) = 1/(2p)$), and of $\left|\bar{B}^0_{phys}\right\rangle$, an initially pure $\bar{B}^0$ ($a_L(0) = -a_H(0) = 1/(2q)$), is given by [27]

$$\begin{aligned} \left|B^0_{phys}(t)\right\rangle =& g_+(t)\left|B^0\right\rangle + (q/p)g_-(t)\left|\bar{B}^0\right\rangle, \\ \left|\bar{B}^0_{phys}(t)\right\rangle =& (p/q)g_-(t)\left|B^0\right\rangle + g_+(t)\left|\bar{B}^0\right\rangle, \end{aligned} \tag{3.15}$$

where $M \equiv \frac{1}{2}(M_H + M_L)$,

$$\begin{aligned} g_+(t) =& e^{-iMt} e^{-\frac{1}{2}\Gamma t} \cos(\tfrac{1}{2}\Delta M\ t), \\ g_-(t) =& e^{-iMt} e^{-\frac{1}{2}\Gamma t} i \sin(\tfrac{1}{2}\Delta M\ t). \end{aligned} \tag{3.16}$$

The mass difference between the two neutral B mesons is measured to be

$$x_d \equiv \Delta M_B / \Gamma_B = 0.67 \pm 0.10. \tag{3.17}$$

The calculation of q/p in the B system is quite different from the K system. Here we expect *model independently*

$$\Delta\Gamma_B \ll \Delta M_B. \tag{3.18}$$

The model independent argument for the relation (3.18) goes as follows [15]. On the one hand, there is the experimental measurement (3.17). On the other hand, $\Delta\Gamma$ has not been measured and is probably impossible to measure. But $\Delta\Gamma$ is produced by decay channels which are common to B^0 and $\bar{B}^0$. The (upper bounds on) branching ratios for such channels are at or below the level of 10^{-3}. As various channels contribute to Γ_{12} with differing signs, one expects that their sum would not exceed the individual level, say

$$\Delta\Gamma_B / \Gamma_B \lesssim 10^{-2}. \tag{3.19}$$

Eqs. (3.17) and (3.19) lead to (3.18) which implies, in turn, $|\Gamma_{12}| \ll |M_{12}|$. Therefore, in the B system

$$\frac{q}{p} \approx -\frac{M_{12}^*}{|M_{12}|}\left[1 - \tfrac{1}{2}\mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right)\right]. \tag{3.20}$$

Note that q/p is a pure phase, up to corrections $\leq \mathcal{O}(10^{-2})$. However, to study the deviation from a pure phase, one needs to calculate Γ_{12} and M_{12}. This will involve large hadronic uncertainties, in particular in the hadronization models for Γ_{12}. In ref. [15] it is estimated that this will induce an overall uncertainty of a factor of 2-3 in $|q/p| - 1$.

4. Experimental Observations of CP Violation

4.1. $|\bar{A}/A| \neq 1$

In the decays of neutral mesons, effects of CP violation in mixing are unavoidable. Thus, to unambiguously observe direct CP violation, it is best to measure CP asymmetries in charged meson decays,

$$a_f = \frac{\Gamma(P^+ \to f) - \Gamma(P^- \to \bar{f})}{\Gamma(P^+ \to f) + \Gamma(P^- \to \bar{f})}. \tag{4.1}$$

In terms of decay amplitudes

$$a_f = \frac{1 - |\bar{A}/A|^2}{1 + |\bar{A}/A|^2}. \tag{4.2}$$

As discussed above, $a_f \neq 0$ requires contributions to the decay process which differ in both their strong phases and their weak phases so that $|\bar{A}/A| \neq 1$. Purely leptonic and semileptonic decays are dominated by a single diagram and thus are unlikely to exhibit any measurable direct CP violation. On the other hand, nonleptonic decays have often contributions from at least two types of processes. This has to do with the existence of tree and penguin processes. The two types of diagrams are depicted in Fig. 1.

In penguin processes there is a loop with a W boson, while all other processes of order G_F are tree processes. Penguin diagrams can be further classified according to the identity of the quark in the loop, as diagrams with different intermediate quarks may have both different strong phases and different weak phases. On the other hand, the subdivision of tree processes into spectator, exchange and annihilation diagrams is unimportant since they all carry the same weak phase.

There are three particularly promising types of processes [28]:

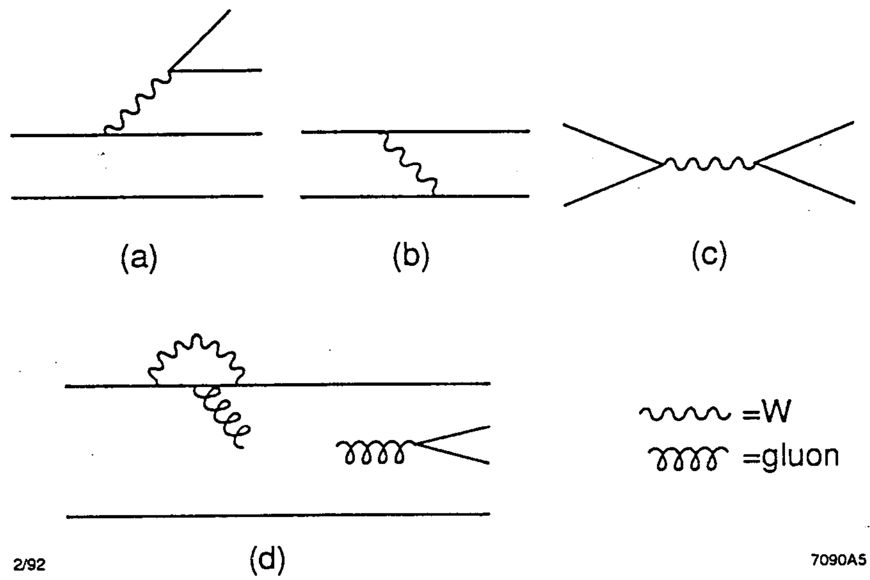

Figure 1. Meson decays relevant to our discussion divide into tree diagrams ((a) spectator, (b) exchange, (c) annihilation) and (d) penguin diagrams. Penguin diagrams may contain any number of gluons between the quark lines, but if perturbative QCD holds, the leading contribution comes from a diagram where the two gluon lines in (d) are connected.

a. Decays with suppressed tree contribution. In this type of decays, the penguin and the tree contributions may be comparable in magnitude and give large interference effects. An example is the decay $B \to \rho K$ (where the tree decay is suppressed by small mixing angles, $V_{ub}V_{us}$).

b. Decays with forbidden tree decays. Here the interference may come from penguin contributions with different charge 2/3 quarks in the loop. Examples are $B \to \phi K$ and $B \to KK$.

c. Radiative decays. The mechanism here is the same as in case *b* except that the leading contribution to the decay is an electromagnetic penguin.

It is unfortunate that the leading nonleptonic K decay, $K^{\pm} \to \pi^{\pm}\pi^{0}$, is unlikely to have direct CP violation. The reason is as follows. The K^{+} meson is a member in an isospin doublet, $I = 1/2$. The final $\pi^{+}\pi^{0}$ state has $I_3 = 1$, and from Bose symmetry it cannot be an $I = 1$ state and therefore must be $I = 2$. Consequently, the decay has only one isospin channel, $\Delta I = 3/2$. As strong interactions are isospin invariant, there is only one strong phase shift, denoted by δ_2. The condition of contributions from different strong phases is not met and

$$a_{\pi^+\pi^0} = 0. \tag{4.3}$$

The same argument holds for $B^\pm \to \pi^\pm\pi^0$.

There is no unambiguous experimental evidence for direct CP violation yet.

4.2. $|q/p| \neq 1$

We now study the decays $P^0, \bar{P}^0 \to \ell^\pm \nu X$. From the $\Delta P = \Delta Q$ rule,

$$P^0 \not\to \ell^- \nu X, \quad \bar{P}^0 \not\to \ell^+ \nu X. \tag{4.4}$$

For the allowed processes, we define the following amplitude:

$$\langle \ell^+ \nu X | \, H \, | P^0 \rangle = A, \quad \langle \ell^- \nu X | \, H \, | \bar{P}^0 \rangle = A^*. \tag{4.5}$$

For the K system, we can measure

$$a_{sl} = \frac{\Gamma(K_L \to \ell^+ \nu X) - \Gamma(K_L \to \ell^- \nu X)}{\Gamma(K_L \to \ell^+ \nu X) + \Gamma(K_L \to \ell^- \nu X)}. \tag{4.6}$$

As

$$\langle \ell^+ \nu X | \, H \, | K_L \rangle = pA, \quad \langle \ell^- \nu X | \, H \, | K_L \rangle = qA^*, \tag{4.7}$$

we get

$$a_{sl} = \frac{1 - |q/p|^2}{1 + |q/p|^2}. \tag{4.8}$$

With the notation (2.5), (4.8) becomes $a_{sl} = 2\mathrm{Re}(\bar{\epsilon})/(1 + |\bar{\epsilon}|^2)$.

a_{sl} was measured for both final e and final μ. The weighted average is [26]

$$a_{sl} = (3.27 \pm 0.12) \times 10^{-3}. \tag{4.9}$$

For the B system, we can measure

$$a_{sl} = \frac{\Gamma(\bar{B}^0_{phys}(t) \to \ell^+\nu X) - \Gamma(B^0_{phys}(t) \to \ell^-\nu X)}{\Gamma(\bar{B}^0_{phys}(t) \to \ell^+\nu X) + \Gamma(B^0_{phys}(t) \to \ell^-\nu X)}. \tag{4.10}$$

As

$$\langle \ell^-\nu X | H | B^0_{phys}(t) \rangle = (q/p) g_-(t) A^*, \quad \langle \ell^+\nu X | H | \bar{B}^0_{phys}(t) \rangle = (p/q) g_-(t) A, \tag{4.11}$$

we get

$$a_{sl} = \frac{1 - |q/p|^4}{1 + |q/p|^4}. \tag{4.12}$$

There is no experimental measurement yet of a_{sl} in B decays.

For both the K and the B systems, the CP asymmetry in semileptonic decay depends on the deviation of $|q/p|$ from unity.

4.3. $\lambda \neq 1$

The importance of CP violation in neutral meson decays into final CP eigenstates lies in the possibility of theoretical interpretation free of hadronic uncertainties. Moreover, the two CP violating parameters which have been experimentally measured, ϵ and ϵ'/ϵ, belong to this class of CP violation.

The two CP violating quantities measured in the neutral K system are

$$\eta_{00} = \frac{\langle \pi^0\pi^0 | H | K_L \rangle}{\langle \pi^0\pi^0 | H | K_S \rangle}, \quad \eta_{+-} = \frac{\langle \pi^+\pi^- | H | K_L \rangle}{\langle \pi^+\pi^- | H | K_S \rangle}. \tag{4.13}$$

The experimental results are [26]

$$|\eta_{00}| = (2.253 \pm 0.024) \times 10^{-3}, \quad \phi_{00} = 46.6 \pm 2.0^o;$$
$$|\eta_{+-}| = (2.268 \pm 0.023) \times 10^{-3}, \quad \phi_{+-} = 46.6 \pm 1.2^o. \tag{4.14}$$

We define

$$A_{00} = \langle \pi^0\pi^0 | H | K^0 \rangle, \quad \bar{A}_{00} = \langle \pi^0\pi^0 | H | \bar{K}^0 \rangle,$$
$$A_{+-} = \langle \pi^+\pi^- | H | K^0 \rangle, \quad \bar{A}_{+-} = \langle \pi^+\pi^- | H | \bar{K}^0 \rangle, \tag{4.15}$$

$$\lambda_{00} \equiv \frac{q}{p}\frac{\bar{A}_{00}}{A_{00}}, \quad \lambda_{+-} \equiv \frac{q}{p}\frac{\bar{A}_{+-}}{A_{+-}}. \tag{4.16}$$

Then

$$\eta_{00} = \frac{pA_{00} - q\bar{A}_{00}}{pA_{00} + q\bar{A}_{00}} = \frac{1-\lambda_{00}}{1+\lambda_{00}},$$
$$\eta_{+-} = \frac{pA_{+-} - q\bar{A}_{+-}}{pA_{+-} + q\bar{A}_{+-}} = \frac{1-\lambda_{+-}}{1+\lambda_{+-}}. \tag{4.17}$$

As we shall later see in detail, η_{00} and η_{+-} are affected by all three types of CP violation: $|q/p| \neq 1$ and $\mathrm{Im}\lambda \neq 0$ give $\mathcal{O}(10^{-3})$ effects, while $|\bar{A}/A| \neq 1$ gives an $\mathcal{O}(10^{-6})$ effect.

For the B system, we should measure quantities of the form [6, 29, 30]

$$a_{f_{CP}} \equiv \frac{\Gamma(B^0_{\mathrm{phys}}(t) \to f_{CP}) - \Gamma(\bar{B}^0_{\mathrm{phys}}(t) \to f_{CP})}{\Gamma(B^0_{\mathrm{phys}}(t) \to f_{CP}) + \Gamma(\bar{B}^0_{\mathrm{phys}}(t) \to f_{CP})}. \tag{4.18}$$

Eqs. (3.15) and (3.16) lead to the following form for the time-dependent asymmetry:

$$a_{f_{CP}} = \frac{(1-|\lambda|^2)\cos(\Delta M t) - 2\mathrm{Im}\lambda \sin(\Delta M t)}{1+|\lambda|^2}. \tag{4.19}$$

For decay modes such that $|\lambda| = 1$ (the "clean" modes), (4.19) simplifies considerably:

$$a_{f_{CP}}(|\lambda| = 1) = -\mathrm{Im}\lambda \sin(\Delta M t). \tag{4.20}$$

The modes appropriate for measuring asymmetries of the type (4.20) are those dominated by a single weak phase. Likely candidates are ψK_S, D^+D^-, $\pi^+\pi^-$, ϕK_S and others.

5. Theoretical Calculations of *CP* Violation

In this chapter we point out the hadronic uncertainties that enter the calculations of *CP* violating phenomena. The reason for hadronic uncertainties is that we do not understand low energy strong interactions in quantitative detail. We separate our calculations into two parts. First, we calculate the effective Lagrangian in terms of quark and gluon fields at a high energy scale, typically $\sim m_Z$, and use Renormalization Group Equations (RGEs) to run $\mathcal{L}_{eff}$ down to the relevant hadronic scale. This part is well understood and can be calculated with high accuracy. Second, to calculate physical decay rates (or mixing) we must calculate the matrix elements of $\mathcal{L}_{eff}$ between the relevant physical states. That is the part where we lack in theoretical technology. In some cases, *e.g.* semileptonic meson decays, approximate symmetries may help us fix the form and normalization of the matrix element. Known examples are the chiral symmetry for K decay and heavy quark symmetry for B decay. However, in nonleptonic decays (and in mixing amplitudes) the quark operators do not correspond to currents and therefore we do not know the normalization of their matrix elements. We may use phenomenological models to estimate them but have little control over the resulting uncertainties. Eventually, lattice calculations may solve the problem, but at present they are also subject to approximations and uncertainties.

There is a significant difference in the cleanliness of the theoretical calculations in the three types of *CP* violation. Furthermore, there are differences in the cleanliness of predictions for *CP* violating quantities between the K and the B systems. In this chapter we clarify these issues.

From eqs. (2.3), (2.1) and (2.6), we see that the relevant quantities that need to be calculated are q/p and $\bar{A}/A$. Let us start with the latter one. Recall eq.

(1.2):

$$A = \sum_i A_i e^{i\delta_i} e^{i\phi_i}; \quad \bar{A} = e^{-2i\xi_P} e^{+2i\xi_f} \sum_i A_i e^{i\delta_i} e^{-i\phi_i}. \tag{5.1}$$

Notice the following two facts:

a. If all contributing amplitudes had the same strong phase shift, then $\bar{A}/A$ would be a pure phase.

b. If all contributing amplitudes had the same weak phase, then $\bar{A}/A$ would be a pure phase.

Thus, for *direct CP* violation, $|\bar{A}/A| \neq 1$, there should be both non-trivial CP conserving phases ($\delta_i - \delta_j \neq 0$) and non-trivial CP violating phases ($\phi_i - \phi_j \neq 0$). Conversely, the calculation of direct CP violation requires knowledge of strong phase shifts and of absolute values of various amplitudes and therefore necessarily involves hadronic uncertainties.★

In the previous sections we concluded that for both the K and the B systems, q/p is of the form $q/p = e^{i\phi}(1 + x)$, where ϕ is a phase which depends purely on phase convention and electroweak parameters, and x is small, $\mathcal{O}(10^{-3})$, but has hadronic uncertainties. In the K system these uncertainties arise from the B_K parameter in the calculation of M_{12}. In the B system the uncertainties arise from the need to calculate Γ_{12}. But in any case, we are led to one conclusion for both systems: effects of CP violation in mixing, namely $|q/p| \neq 1$, are small and subject to large hadronic uncertainties for both K^0 and B^0.

This leaves one possibility for a potentially clean CP violating quantity, namely CP violation in the interference of mixing and decay. The condition is that we have to choose decays into final CP eigenstates which are dominated by a single CP violating phase. Then $\bar{A}_{f_{CP}}/A_{f_{CP}}$ is a pure phase with no hadronic uncertainties. Such modes are available in principle for both K and B. For

★ In some cases, it is possible to overcome the hadronic uncertainties by measuring several isospin-related rates [31, 32, 33].

K^0 decays, we look into either $\pi^+\pi^-$ or $\pi^0\pi^0$. The $\Delta I = 1/2$ rule implies that both are dominated by a single strong phase δ_0. For B^0 decays we may choose, for example, ψK_S. It is dominated by a single weak phase. Then, in principle, the phase difference between (q/p) (neglecting the small deviation from a pure phase) and $(\bar{A}/A)$ will determine the CP asymmetry and is free of hadronic uncertainties!

In practice this observation is useful only in the B system. The reason that it does not work in the K system is that the difference in width, Γ_{12}, is completely dominated by the two pion intermediate state and therefore

$$\arg(\Gamma_{12}) = \arg(A_{2\pi}^* \bar{A}_{2\pi}) = \arg(\bar{A}_{2\pi}/A_{2\pi}). \tag{5.2}$$

In the approximation that $(\bar{A}_{2\pi}/A_{2\pi})$ is a pure phase we consequently have

$$\frac{\bar{A}_{2\pi}}{A_{2\pi}} = -\frac{\Delta\Gamma}{2\Gamma_{12}^*} = e^{-2i\xi}. \tag{5.3}$$

(See eq. (3.10) for the last equation.) However, eq. (3.11) shows that in the approximation where q/p is a pure phase it is given by $q/p = e^{2i\xi}$. Thus, the prediction for CP asymmetry in $K \to 2\pi$ which is clean of hadronic uncertainties is simply zero:

$$\lambda(K \to \pi\pi) = 1 \Longrightarrow \mathrm{Im}\lambda_{\pi\pi} = 0. \tag{5.4}$$

It should hold (as it does!) up to $\mathcal{O}(10^{-3})$. To learn something about CP violation we have to give up this approximation and use

$$\frac{q}{p}\frac{\bar{A}_{\pi\pi}}{A_{\pi\pi}} = 1 - i\phi_{12}\frac{1 + i\frac{\Delta\Gamma}{2\Delta M}}{1 + \left(\frac{\Delta\Gamma}{2\Delta M}\right)^2}. \tag{5.5}$$

Therefore, we would encounter hadronic uncertainties.

On the other hand, to take (q/p) of the B system to be a pure phase means that we set $|\Gamma_{12}/M_{12}| \to 0$. The phase of Γ_{12} or, more important, of any exclusive CP eigenmode, is still different from that of M_{12} and we may have (as we do!) clean predictions for large CP asymmetries in the decays of neutral B into CP eigenstates.

6. The ϵ and ϵ' Parameters

6.1. WHAT ARE ϵ AND ϵ'/ϵ?

There is a possible contribution in (4.17) from direct CP violation [34, 35]. This is due to the fact that there are two isospin channels, leading to final $(2\pi)_{I=0}$ and $(2\pi)_{I=2}$ states:

$$\begin{aligned} \langle\pi^0\pi^0| &= \sqrt{\tfrac{1}{3}}\,\langle(\pi\pi)_{I=0}| - \sqrt{\tfrac{2}{3}}\,\langle(\pi\pi)_{I=2}|\,, \\ \langle\pi^+\pi^-| &= \sqrt{\tfrac{2}{3}}\,\langle(\pi\pi)_{I=0}| + \sqrt{\tfrac{1}{3}}\,\langle(\pi\pi)_{I=2}|\,. \end{aligned} \qquad (6.1)$$

However, the possible effects are small because (on top of the smallness of all CP violating phases in the K system) the final $I = 0$ state is dominant (this is the $\Delta I = 1/2$ rule). Defining

$$A_I = \langle(\pi\pi)_I|\, H\, |K^0\rangle\,, \quad \bar{A}_I = \langle(\pi\pi)_I|\, H\, |\bar{K}^0\rangle\,, \qquad (6.2)$$

we have, experimentally,

$$|A_2/A_0| \approx 1/20. \qquad (6.3)$$

Instead of η_{00} and η_{+-} we may define two combinations, ϵ and ϵ', in such a way that the possible direct CP violating effects are isolated into ϵ'.

Our experimental definition of the ϵ parameter is then:

$$\epsilon \equiv \tfrac{1}{3}(\eta_{00} + 2\eta_{+-}). \qquad (6.4)$$

To zeroth order in A_2/A_0, we have $\eta_{00} = \eta_{+-} = \epsilon$. However, the specific combination (6.4) is chosen in such a way that the following relation holds to *first* order in A_2/A_0 (see (6.1)):

$$\epsilon = \frac{1-\lambda_0}{1+\lambda_0}. \qquad (6.5)$$

As, by definition, only one strong channel contributes to λ_0, there is indeed no direct CP violation in (6.5). Eq. (6.5) may serve as a theoretical definition of ϵ. The two definitions, (6.4) and (6.5), are identical to an excellent approximation.

Is ϵ a manifestation of CP violation in mixing or in the interference of mixing and decay? The answer is that in the K system the two are related, and thus $\epsilon \neq 0$ is a manifestation of both. To be explicit, we examine eqs. (3.11) and (5.5):

$$\begin{aligned} \left|\frac{q}{p}\right| - 1 &= 2\phi_{12}\, \frac{\frac{\Delta\Gamma}{2\Delta M}}{1 + \left(\frac{\Delta\Gamma}{2\Delta M}\right)^2}, \\ 2\epsilon \approx 1 - \frac{q}{p}\,\frac{\bar{A}_0}{A_0} &= \phi_{12}\frac{i - \frac{\Delta\Gamma}{2\Delta M}}{1 + \left(\frac{\Delta\Gamma}{2\Delta M}\right)^2}. \end{aligned} \qquad (6.6)$$

As $\Delta\Gamma \approx -2\Delta M$, the deviation of $|q/p|$ from 1 (CP violation in mixing) and the phase deviation of $(q/p)(\bar{A}_0/A_0)$ from 1 (CP violation in the interference of mixing and decay) are both $\mathcal{O}(\phi_{12})$ and thus contribute to ϵ at the same order. One may say that $\mathrm{Re}(\epsilon) \neq 0$ is a manifestation of CP violation in mixing, but that (6.6) predicts $\arg(\epsilon) \approx \pi/4$ and so there is also CP violation in the interference of mixing and decay. It is amusing to note that, if $\Delta\Gamma \ll \Delta M$ then ϵ would be a manifestation of interference between mixing and decay only.

Our experimental definition of the ϵ' parameter is

$$\epsilon' = \tfrac{1}{3}(\eta_{+-} - \eta_{00}). \qquad (6.7)$$

Thus

$$\epsilon' = \frac{2(\lambda_{00} - \lambda_{+-})}{3(1+\lambda_{00})(1+\lambda_{+-})} \approx \frac{1}{6}\,\frac{q}{p}\left(\frac{\bar{A}_{00}}{A_{00}} - \frac{\bar{A}_{+-}}{A_{+-}}\right), \tag{6.8}$$

where in the last equality we used (4.14) which gives $\lambda_{+-} \approx \lambda_{00} \approx 1$. We can further evaluate (6.8) in terms of A_0 and A_2. We use $(q/p)(\bar{A}_0/A_0) \approx 1$, as discussed in chapter 5, and $|A_2| \ll |A_0|$ and get

$$\epsilon' = \frac{i}{\sqrt{2}}|A_2/A_0|e^{i(\delta_2-\delta_0)}\sin(\phi_2 - \phi_0). \tag{6.9}$$

As in the derivation of (6.9) we find that replacing q/p with a pure phase is a good approximation, there is no CP violation in mixing in ϵ'. We can now ask whether ϵ' is a manifestation of CP violation in decay or in the interference between mixing and decay. To answer that, we note that $\epsilon' \neq 0$ does not require $\delta_2 \neq \delta_0$. In this sense, $|\epsilon'| \neq 0$ is not a proof of direct CP violation, but $\mathrm{Re}(\epsilon') \neq 0$ is.

The definitions of ϵ in eq. (6.4) and ϵ' in eq. (6.7) give

$$\eta_{+-} = \epsilon + \epsilon', \quad \eta_{00} = \epsilon - 2\epsilon'. \tag{6.10}$$

The way in which ϵ' is determined is actually by measuring

$$|\eta_{00}/\eta_{+-}| \approx 1 - 3\mathrm{Re}(\epsilon'/\epsilon). \tag{6.11}$$

The experimental result is [26]

$$|\eta_{00}/\eta_{+-}| = 0.9935 \pm 0.0032. \tag{6.12}$$

Actually, there are two recent measurements with somewhat different results

[36, 37]:

$$\mathrm{Re}(\epsilon'/\epsilon) = \begin{cases} (2.3 \pm 0.7) \times 10^{-3} & \text{NA31}, \\ (0.6 \pm 0.7) \times 10^{-3} & \text{E731}. \end{cases} \tag{6.13}$$

From eq. (6.6) and using $-\Delta\Gamma/(2\Delta M) \approx 1$, we have

$$\arg(\epsilon) \approx \arctan(-2\Delta M/\Delta\Gamma) = 43.67 \pm 0.13^o. \tag{6.14}$$

From eq. (6.9) and the experimental values of δ_2 and δ_0, we have

$$\arg(\epsilon') = \pi/2 + \delta_2 - \delta_0 \approx 47 \pm 5^o. \tag{6.15}$$

Thus, we get $\arg(\epsilon'/\epsilon) \approx 0$. Then

$$\mathrm{Re}(\epsilon'/\epsilon) \approx \epsilon'/\epsilon. \tag{6.16}$$

This is why you may often encounter statements that the ratio (6.11) gives a measurement of ϵ'/ϵ.

6.2. HADRONIC UNCERTAINTIES IN THE CALCULATION OF ϵ

In a phase convention for the K system where all phases are small, and using $\Delta\Gamma \approx -2\Delta M$, we may write

$$\begin{aligned} \frac{q}{p} &= -\frac{2[\mathrm{Re}M_{12}(1+i) - i\mathrm{Im}M_{12} - \frac{1}{2}\mathrm{Im}\Gamma_{12}]}{\Delta M(1+i)} \\ &\approx e^{2i\xi} + \frac{1}{(1+i)}\left(\frac{i\mathrm{Im}M_{12}}{\Delta M} - \frac{\mathrm{Im}\Gamma_{12}}{\Delta\Gamma}\right). \end{aligned} \tag{6.17}$$

In the limit of CP invariance, $q/p = e^{2i\xi}$ so that K_S (K_L) is a pure CP even (odd) state. In the notation (1.20), (6.17) translates into

$$\bar{\epsilon} = \frac{1}{(1+i)}\left(\frac{i\mathrm{Im}M_{12}}{\Delta M} - \frac{\mathrm{Im}\Gamma_{12}}{\Delta\Gamma}\right). \tag{6.18}$$

To calculate the last term, we use the fact that for the K system Γ_{12} is dominated by the intermediate $(\pi\pi)_{I=0}$ state. Eq. (1.5) gives then

$$\mathrm{Im}\Gamma_{12}/\mathrm{Re}\Gamma_{12} = \mathrm{Im}(a_0^*)^2/\mathrm{Re}(a_0^*)^2 \approx -2\mathrm{Im}(a_0)/\mathrm{Re}(a_0) \equiv -2t_0, \tag{6.19}$$

where a_0 is the amplitude for neutral K decay into two pions in isospin-zero state, with the strong phase shift factored out:

$$\langle(\pi\pi)_I|\, H\, |K^0\rangle = a_I e^{i\delta_i}, \quad \langle(\pi\pi)_I|\, H\, |\bar{K}^0\rangle = a_I^* e^{i\delta_i}. \tag{6.20}$$

The quantity t_0 has an upper bound from measurements of the ϵ'/ϵ parameter to be discussed later. This bound implies that the it is the first term in the parenthesis in the RHS of (6.17) which dominates. The main theoretical input is then in the calculation of M_{12}. There are two main uncertainties in this calculation:

a. Long distance contributions. These are parametrized by a parameter D,

$$D = \frac{(M_{12})_{LD}}{M_{12}}. \tag{6.21}$$

The intermediate states that contribute to $(M_{12})_{LD}$ include π^0, η, 2π, 3π, η' and others. It is important to realize that long distance processes contribute differently to ImM_{12} and to ReM_{12} (see the clear discussion in ref. [38] and references therein). The contribution to ReM_{12} could be significant: all the states mentioned above could contribute to ΔM at the same order or even dominate over the short distance contributions, namely D of order 1 is not unlikely. On the other hand, it is commonly believed that the long distance contributions are not important in $\bar{\epsilon}$. All the dispersive diagrams involving π^0, η, 2π and 3π share the same phase because their amplitudes are related by PCAC and $SU(3)$, and the PCAC extrapolation is the same for CP conserving and CP violating interactions. These contributions all obey the relation

$$\frac{\mathrm{Im}(M_{12})_{\pi,\eta_8,2\pi,3\pi}}{\Delta M} = -D' t_0, \tag{6.22}$$

where D' is the contribution to D from these states. The contribution from an intermediate η_0 could be important and does not obey (6.22). Still, it is proportional to t_0,

$$\frac{\text{Im}(M_{12})_{\eta_0}}{\Delta M} = N_{\eta_0} t_0. \tag{6.23}$$

Calculations of N_{η_0} are model dependent but do not show any surprising enhancement, $N_{\eta_0} \leq 1$. Thus, as long as neither D' nor N_{η_0} are particularly large, long distance contributions to ImM_{12} are small, while for ReM_{12} they may be large.

b. The vacuum insertion approximation. The short distance contributions depend on a matrix element of a four quark operator between K^0 and $\bar{K}^0$ states. At present, there is no model independent way to calculate it. We parametrize this uncertainty with a parameter B_K, which is just the ratio between the true value of the matrix element and its value in the vacuum insertion approximation:

$$B_K \equiv \frac{\left\langle K^0 \right| \bar{d}\gamma_\mu(1-\gamma_5)s\bar{d}\gamma^\mu(1-\gamma_5)s \left| \bar{K}^0 \right\rangle}{\left\langle K^0 \right| \bar{d}\gamma_\mu(1-\gamma_5)s \left| 0 \right\rangle \left\langle 0 \right| \bar{d}\gamma^\mu(1-\gamma_5)s \left| \bar{K}^0 \right\rangle}. \tag{6.24}$$

Note that B_K affects $\text{Im}(M_{12})_{SD}$ and $\text{Re}(M_{12})_{SD}$ in the same way.

If D were small, then we would calculate $\text{Im}(M_{12})/\text{Re}(M_{12})$ taking into account only short distance contributions. In this case, B_K would cancel out of the calculation and the hadronic uncertainties would be negligible. However, D is probably not small and, furthermore, we have no reliable way to calculate it. Thus we prefer to use $\text{Im}(M_{12})/\Delta M$ which, though independent of D, has large uncertainties from B_K.

7. Summary

There are three types of CP violation in meson decays:

(i) $|\bar{A}/A| \neq 1$.

$$\left|\frac{\bar{A}}{A}\right| = \left|\frac{\sum_i A_i e^{i\delta_i} e^{-i\phi_i}}{\sum_i A_i e^{i\delta_i} e^{+i\phi_i}}\right|. \tag{7.1}$$

CP violation results from interference between direct decay amplitudes. It can be observed in nonleptonic charged meson decays. There are large hadronic uncertainties in the calculation.

(ii) $|q/p| \neq 1$.

$$\left|\frac{q}{p}\right| = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}. \tag{7.2}$$

CP violation results from the physical neutral meson states being different from the CP eigenstates. It can be observed in semileptonic neutral meson decays. There are hadronic uncertainties in the calculation.

(iii) $\lambda \neq 1$.

$$\lambda = \left(\frac{q}{p}\right)\left(\frac{\bar{A}_{f_{CP}}}{A_{f_{CP}}}\right). \tag{7.3}$$

CP violation with $|\lambda| = 1$, $\mathrm{Im}\lambda \neq 0$, results from interference between mixing and decay. It can be observed in neutral meson decays into CP eigenstates. There exist several B decay modes that have only tiny hadronic uncertainties in the calculation.

I′. ELECTRIC DIPOLE MOMENTS (EDMs)

8. Why Are EDMs CP Violating

An electric dipole moment (EDM) $\mathcal{D}$ of an elementary particle is a manifestation of CP violation [39]. The simple argument for that is as follows. The only vector which characterizes an elementary particle is its spin J_i. Therefore, we must have

$$\mathcal{D}_i = dJ_i. \tag{8.1}$$

Under P-transformation $\mathcal{D} \to -\mathcal{D}$ and $J \to J$. Under T-transformation $\mathcal{D} \to \mathcal{D}$ and $J \to -J$. Consequently, if either P or T (or, equivalently, CP) is a good symmetry, we must have $d = 0$. A more formal proof goes as follows [40]. Let us study the matrix element of $\mathcal{D}_0$ for a state with spin S:

$$\langle S\ M|\, \mathcal{D}_0\, |SM\rangle = ||\mathcal{D}|| \begin{pmatrix} S & S & 1 \\ M & -M & 0 \end{pmatrix}. \tag{8.2}$$

Using T invariance we get

$$\begin{aligned} \langle S\ M|\, T^{-1}T\mathcal{D}_0T^{-1}T\, |S\ -M\rangle =& (-1)^{2M} \langle S\ -M|\, \mathcal{D}_0\, |S\ -M\rangle \\ =& (-1)^{2M+2S+1} ||\mathcal{D}|| \begin{pmatrix} S & S & 1 \\ M & -M & 0 \end{pmatrix}. \end{aligned} \tag{8.3}$$

Using $S + M =$ integer ($\Rightarrow (-1)^{2(M+S)+1} = -1$) we conclude that $||\mathcal{D}|| = 0$.

Most of our discussion of EDMs will concentrate in the EDM of the neutron, $\mathcal{D}_n$. One may wonder why should the above argument apply to it, as the neutron is not an elementary particle. The answer is that the only feature of the particle that we used in (8.1) is that it is characterized by its spin only. This certainly applies to the neutron as well. (Otherwise, there should have been degenerate neutron states.)

No EDM of an elementary particle has been observed yet. The most useful upper bound (for our purposes) is that on the EDM of the neutron [10, 11],

$$|\mathcal{D}_n| \leq 1.2 \times 10^{-25}\ e\ \text{cm}. \tag{8.4}$$

We also use the upper bound on the EDM of the electron [41],

$$|\mathcal{D}_e| \leq 1.5 \times 10^{-26}\ e\ \text{cm}. \tag{8.5}$$

9. Hadronic Uncertainties in $\mathcal{D}_n$

The current experimental bound on the EDM of the neutron (8.4) provides one of the most sensitive constraints on CP violating extensions of the Standard Model. However, the strong interactions are an obstacle to improving the constraints from $\mathcal{D}_n$. The essential problem is to calculate the neutron dipole moment induced by a given CP violating operator, where the operator is generated by short distance physics and is expressed in terms of quark and gluon fields. In some cases, it is possible to make a current algebra calculation of contributions that diverge in the chiral limit [9] so that they are formally dominant, but for most operators one has to resort to a non-relativistic approximation [42] or simply to a naive dimensional analysis [43 – 45]. Lattice calculations are still far from practicality [46]. We discuss three useful examples: current algebra calculation of the contribution from a two gluon operator, non-relativistic approximation for a two quark operator, and naive dimensional estimate of a three gluon operator.

A two gluon operator of the form

$$\frac{g_s^2}{32\pi^2}\theta G_a^{\mu\nu}\tilde{G}_{a\mu\nu} \tag{9.1}$$

can be transformed, using anomaly relations, into [8] CP violating quark operators:

$$\frac{3m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s}\theta i(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s). \tag{9.2}$$

This can be translated into imaginary part in the mass terms for the meson octet in the chiral lagrangian,

$$\mathcal{L}_M = \frac{F_\pi^2}{16} \operatorname{tr}(MU + M^\dagger U^\dagger - M - M^\dagger), \tag{9.3}$$

with

$$U = \exp\left(\frac{2i}{F_\pi}\phi_a\lambda_a\right). \tag{9.4}$$

The most singular contribution to $\mathcal{D}_n$ in the chiral $m_\pi \to 0$ limit was identified in ref. [9] as coming from a one loop diagram, with the result

$$\mathcal{D}_n = \frac{g_{\pi NN}\bar{g}_{\pi NN}}{4\pi^2 M_N} \ln(M_N/m_\pi) = +3.6 \times 10^{-16}\ \theta\ e \text{ cm}. \tag{9.5}$$

Here M_N is the nucleon mass, and $g_{\pi NN}$ ($\bar{g}_{\pi NN}$) is the pseudoscalar coupling (CP violating scalar coupling) of the nucleon.

A very different approach was taken in ref. [47] where the Skyrme model was used to calculate the contribution from (9.3) to $\mathcal{D}_n$. The results are numerically similar though the calculated contributions are different: ref. [47] has contributions of order $m_\pi^2 N_c$ while ref. [9] calculates contributions of order $m_\pi^2 \ln m_\pi^2$. This implies that the corrections to either result are of $\mathcal{O}(1)$, and they should be taken only as an order of magnitude estimate, namely within a factor of a few.

In many models, it is simple to calculate the EDM of the elementary fields, namely $\mathcal{D}_u$ and $\mathcal{D}_d$ for the up quark and the down quark, respectively. Then, a non-relativistic approximation relates these to the EDM of the nucleon through $SU(6)$ wavefunction relations:

$$\mathcal{D}_n = \tfrac{4}{3}\mathcal{D}_d - \tfrac{1}{3}\mathcal{D}_u, \quad \mathcal{D}_p = \tfrac{4}{3}\mathcal{D}_u - \tfrac{1}{3}\mathcal{D}_d. \tag{9.6}$$

The result for $\mathcal{D}_q$ is proportional to m_q. An instructive measure of the uncertainty in the calculation is the fact that it is not at all clear whether one should use running quark masses at the hadronic scale (say, 1 GeV) or constituent quark masses. The difference for the u and the d quarks is about two orders of magnitude.

There is one dimension six operator that is P and CP violating whose coefficient involves neither light quark masses nor small mixing angles. It is the three gluon operator [45]

$$-\tfrac{1}{6}C f_{abc} G_{a\mu\rho} G^{\rho}_{b\nu} \tilde{G}^{\mu\nu}_{c}. \tag{9.7}$$

A naive dimensional analysis gives a contribution to $\mathcal{D}_n$ of order

$$\mathcal{D}_n \approx \frac{eMC}{4\pi}, \tag{9.8}$$

where $M = 2\pi F_\pi \approx 1190\ MeV$ is the chiral symmetry breaking scale. A typical measure of the uncertainty here is that various analyses may differ by a factor of $(4\pi)^3$, namely by three orders of magnitude.

II. *CP* VIOLATION IN THE STANDARD MODEL

10. The CKM Picture of *CP* Violation

In the Standard Model of $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry with three fermion generations, *CP* violation arises from a single phase in the mixing matrix for quarks. Each quark generation consists of three multiplets:

$$Q_L^I = \begin{pmatrix} U_L^I \\ D_L^I \end{pmatrix} = (3,2)_{1/6}; \quad U_R^I = (3,1)_{2/3}; \quad D_R^I = (3,1)_{-1/3}. \tag{10.1}$$

The interactions of quarks with the $SU(2)_L$ gauge bosons are given by

$$-\mathcal{L}_W = \tfrac{1}{2} g \overline{Q_{Li}^I} \gamma^\mu \tau^a \mathbf{1}_{ij} Q_{Lj}^I W_\mu^a, \tag{10.2}$$

where γ^μ operates in Lorentz space, τ^a operates in $SU(2)_L$ space and $\mathbf{1}$ is the unit matrix operating in generation space. We have written this unit matrix explicitly to make the transformation to mass eigenbasis clearer. The interactions of quarks with the single scalar doublet $\phi(1,2)_{1/2}$ of the Standard Model are given by

$$-\mathcal{L}_Y = G_{ij} \overline{Q_{Li}^I} \phi d_{Rj}^I + F_{ij} \overline{Q_{Li}^I} \tilde{\phi} u_{Rj}^I + \text{h.c.}. \tag{10.3}$$

G and F are general *complex* 3×3 matrices. Their complex nature is the source of *CP* violation in the Standard Model. With the spontaneous symmetry breaking $SU(2)_L \times U(1)_Y \to U(1)_{EM}$ due to $\langle \phi \rangle \neq 0$, the two components of the quark doublet become distinguishable, as are the three members of the W triplet. The charged current interaction in (10.2) is given by

$$-\mathcal{L}_W = \sqrt{\tfrac{1}{2}} g \overline{u_{Li}^I} \gamma^\mu \mathbf{1}_{ij} d_{Lj}^I W_\mu^+ + \text{h.c.}. \tag{10.4}$$

The mass terms that arise from the replacement of $\text{Re}(\phi^0) \to \sqrt{\tfrac{1}{2}}(v + H^0)$ in

(10.3) are given by

$$-\mathcal{L}_M = \sqrt{\tfrac{1}{2}}vG_{ij}\overline{d^I_{Li}}d^I_{Rj} + \sqrt{\tfrac{1}{2}}vF_{ij}\overline{u^I_{Li}}u^I_{Rj} + \text{h.c.}, \tag{10.5}$$

namely

$$M_d = Gv/\sqrt{2}, \quad M_u = Fv/\sqrt{2}. \tag{10.6}$$

The phase information is now contained in these mass matrices. To transform to the mass eigenbasis, we find four unitary matrices such that

$$V_{dL}M_dV^\dagger_{dR} = M^{\text{diag}}_d, \quad V_{uL}M_uV^\dagger_{uR} = M^{\text{diag}}_u, \tag{10.7}$$

where M^{diag}_q are diagonal and real (but V_{qL} and V_{qR} are complex). The charged current interactions (10.4) are given in the mass eigenbasis by

$$-\mathcal{L}_W = \sqrt{\tfrac{1}{2}}g\overline{u_{Li}}\gamma^\mu\bar{V}_{ij}d_{Lj}W^+_\mu + \text{h.c.}. \tag{10.8}$$

(Quark fields with no superscript denote mass eigenstates.) The matrix $\bar{V} = V_{uL}V^\dagger_{dL}$ is the mixing matrix for three quark generations. It is a 3×3 unitary matrix. As such it generally depends on 9 parameters, of which three can be chosen real angles and six are phases. However, we may reduce the number of phases in $\bar{V}$ by a transformation

$$\bar{V} \implies V = P_u\bar{V}P^*_d, \tag{10.9}$$

where P_u and P_d are diagonal phase matrices. This is a legitimate transformation because it amounts to redefining the phases of the quark mass eigenstates:

$$q_{Li} \to (P_q)_{ii}q_{Li}, \quad q_{Ri} \to (P_q)_{ii}q_{Ri}, \tag{10.10}$$

which renders M^{diag}_q unchanged (and, in particular, real). The five phase differences among the elements of P_u and P_d can be chosen to eliminate five phases from $\bar{V}$ in the transformation (10.9), so that V has one unremovable phase. This phase [2] is called the Kobayashi–Maskawa (KM) phase and the mixing matrix [48] is called the Cabibbo–Kobayashi–Maskawa (CKM) matrix.

A similar analysis would show that CP violation cannot arise in this way if there were only two quark generations. A 2×2 unitary matrix ($\bar{V}$) has three phases but there are also three phase differences among the elements of two 2×2 phase matrices (P_u and P_d). Thus all phases can be eliminated from the Lagrangian in the two generation case.

The unremovable phase in the CKM matrix allows possible CP violation. To see that, note that the CP transformation of spinor fields is

$$\psi(x) \to -\eta C \psi^*(\tilde{x}), \quad \bar{\psi}(x) \to -\eta^* \bar{\psi}^*(\tilde{x}) C, \tag{10.11}$$

where η is an arbitrary phase, C is the charge conjugation matrix (fulfilling

$$C\gamma_\mu C^{-1} = -\gamma_\mu^T; \quad -C = C^{-1} = C^T = C^\dagger),$$

and $\tilde{x}^\mu = x_\mu$. The CP transformations of scalar and left-handed currents are then

$$\begin{aligned} \bar{\psi}_i \psi_j \to& \bar{\psi}_j \psi_i, \\ \bar{\psi}_i \gamma^\mu (1-\gamma_5) \psi_j \to& - \bar{\psi}_j \gamma_\mu (1-\gamma_5) \psi_i, \end{aligned} \tag{10.12}$$

where we used

$$(\bar{\psi}_i \Gamma \psi_j)^* = -\bar{\psi}_j (\gamma_0 \Gamma^\dagger \gamma_0) \psi_i. \tag{10.13}$$

Charged vector bosons transform under CP according to

$$W_\mu^\pm(x) \to -W^{\mp\mu}(\tilde{x}). \tag{10.14}$$

Mass terms and gauge interactions can be invariant under (10.12) if the masses and couplings are real. In particular, consider the coupling of $W^\pm$ to quarks. It has the form

$$gV_{ij}\bar{u}_i\gamma_\mu W^{+\mu}(1-\gamma_5)d_j + gV^*_{ij}\bar{d}_j\gamma_\mu W^{-\mu}(1-\gamma_5)u_i. \qquad (10.15)$$

The CP operation interchanges the two terms except that V_{ij} and V^*_{ij} are not interchanged. Thus, CP is a good symmetry only if there is a basis in which all couplings and masses are real.

CP is not necessarily violated in the three generation Standard Model. If two quarks in either sector (up or down) were degenerate, one mixing angle and the phase could be removed from V. Thus CP violation requires

$$(m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_t^2 - m_u^2)(m_b^2 - m_s^2)(m_s^2 - m_d^2)(m_b^2 - m_d^2) \neq 0. \quad (10.16)$$

If the value of any of the three mixing angles is 0 or $\pi/2$, then again the phase is removable. Finally, CP would not be violated if the value of the single phase were 0 or π. These last eight conditions are elegantly incorporated into one, parametrization independent, condition [49]. To find this condition, one notes that unitarity requires that for any choice of i, j, k, l (between 1 and 3)

$$\mathrm{Im}[V_{ij}V^*_{ik}V_{lk}V^*_{lj}] = J \sum_{m,n=1}^{3} \epsilon_{ilm}\epsilon_{jkn}. \qquad (10.17)$$

Then, the conditions on the mixing parameters are simply summarized by

$$J \neq 0. \qquad (10.18)$$

The fourteen conditions incorporated in (10.16) and (10.18) can all be written as a single requirement of the mass matrices in the interaction eigenbasis [49],

$$\mathrm{Im}\{\det[M_d M_d^\dagger, M_u M_u^\dagger]\} \neq 0 \Longleftrightarrow CP \text{ violation}. \qquad (10.19)$$

The quantity J is of great interest in the study of CP violation from the CKM matrix. The maximum value that J may assume is $1/(6\sqrt{3})$, but in reality it is known to be smaller than 10^{-4}, providing a concrete meaning to the notion that CP violation in the Standard Model is small.

The unitarity of the CKM matrix is manifest when using an explicit parametrization. There are various useful ways to parametrize it, but the standard choice [26] is a parametrization due to Chau and Keung [50]:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{10.20}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. In the standard parametrization

$$J = c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13}\sin\delta. \tag{10.21}$$

This explicitly shows the requirement that all mixing angles are different from $0, \pi/2$ and the phase different from $0, \pi$.

The unitarity of the CKM matrix implies various relations among its elements. We will find three of them very useful to our understanding of the Standard Model predictions for CP violation:

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0, \tag{10.22}$$

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0, \tag{10.23}$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \tag{10.24}$$

Each of these three relations requires the sum of three complex quantities to vanish and so can be geometrically represented in the complex plane as a triangle. These are "the unitarity triangles", though the term "unitarity triangle" is usually reserved for the relation (10.24) only (for reasons to be soon understood).

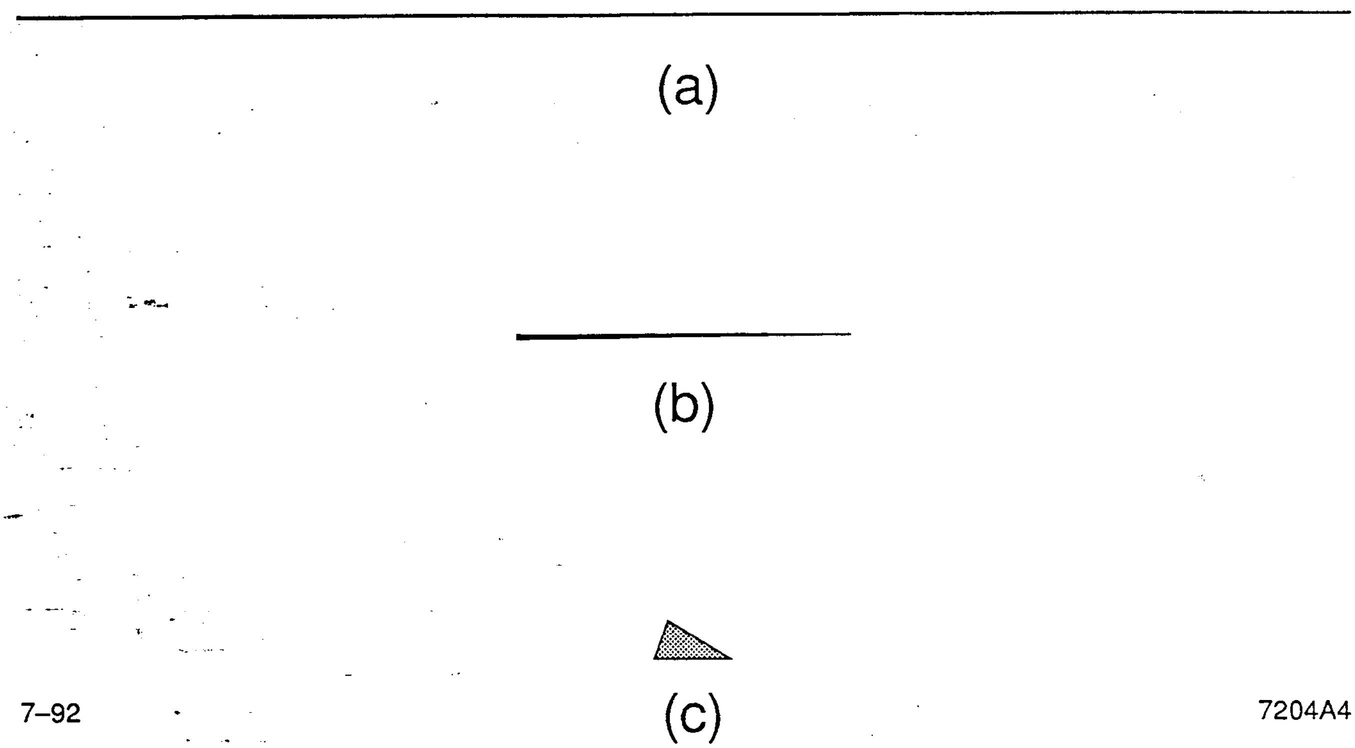

Figure 2. The three unitarity triangles of the CKM matrix: (a) $\sum_i V_{id} V_{is}^* = 0$; (b) $\sum_i V_{is} V_{ib}^* = 0$; (c) $\sum_i V_{id} V_{ib}^* = 0$. The three triangles are drawn at a common scale.

It is instructive to draw the three triangles, knowing the experimental values of the various $|V_{ij}|$. This is done in Fig. 2. In the first two triangles, one side is much shorter than the other two, and so they almost collapse to a line. This would give an intuitive understanding of why CP violation is so small in the K system (the first triangle) and why certain CP asymmetries in B_s decays vanish (the second triangle). The most exciting physics of CP violation lies in the B system, related to the third triangle. Its overall smallness is related to the long lifetime of the B meson. To observe CP asymmetries in B decays, we would have to produce many B^0's because the relevant branching ratios are small. But the openness of the third triangle guarantees that once we produce them, we are likely to observe large CP asymmetries.

Eq. (10.17) has striking implications for the unitarity triangles:

(*i*) All unitarity triangles are equal in area.

(*ii*) The area of each unitarity triangle is given by $\frac{1}{2}|J|$.

(*iii*) The sign of J gives the direction of the complex vectors.

The rescaled unitarity triangle (Fig. 3) is derived from the triangle (10.24) by

a. Choosing a phase convention such that $(V_{cd}V_{cb}^*)$ is real. This aligns one side of the triangle along the real axis.

b. Dividing the lengths of all sides by $|V_{cd}V_{cb}^*|$. This makes the length of the real side 1. The form of the triangle remains unchanged.

Two vertices of the rescaled unitarity triangle are thus fixed at (0,0) and (1,0). The coördinates of the remaining vertex are denoted by (ρ, η) [51]. The three angles of the unitarity triangle are denoted by α, β and γ:

$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right], \quad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right], \quad \gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]. \qquad (10.25)$$

They are physical quantities and, as we will see later, can be independently measured by CP asymmetries in B decays.

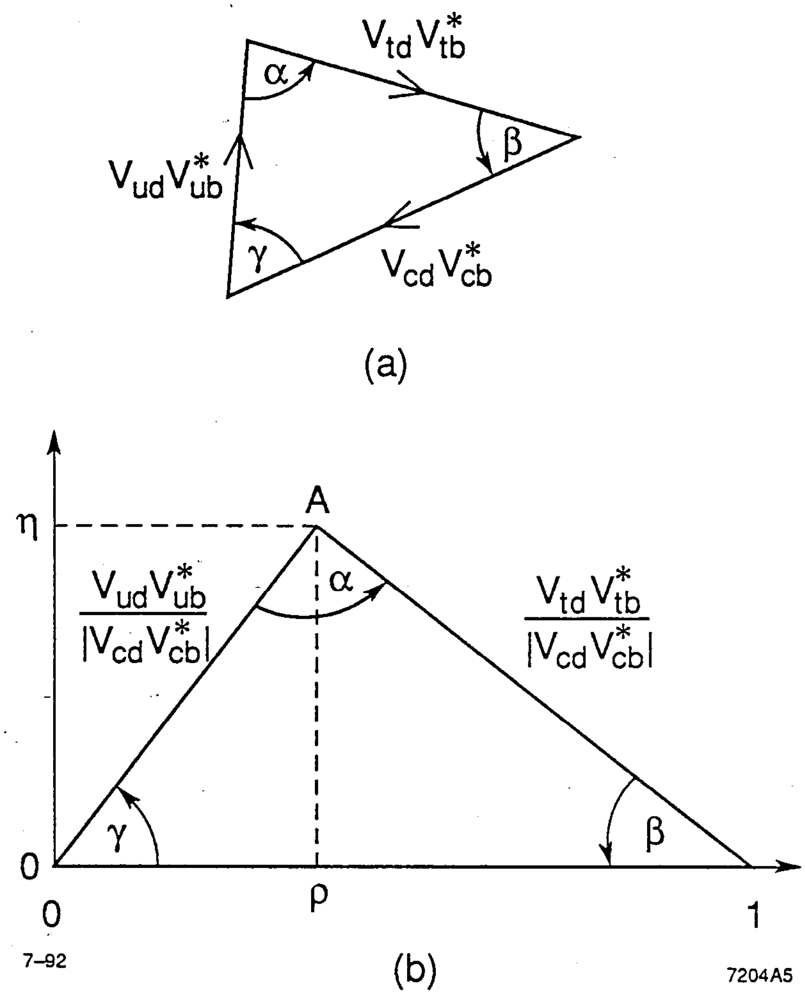

Figure 3. The unitarity triangle $\sum_i V_{id}V_{ib}^* = 0$. (a) shows the original triangle while (b) depicts the rescaled unitarity triangle.

11. Measuring CKM Parameters with *CP* Conserving Processes

Six of the nine absolute values of the CKM entries are measured directly, namely by tree level processes. Nuclear beta decays give

$$|V_{ud}| = 0.9744 \pm 0.0010. \tag{11.1}$$

Semileptonic kaon decays, $K \to \pi e \nu$, and hyperon decays give

$$|V_{us}| = 0.2205 \pm 0.0018. \tag{11.2}$$

Semileptonic D decays, $D \to \pi e \nu$, and neutrino and antineutrino production of charm off valence d quarks give

$$|V_{cd}| = 0.204 \pm 0.017. \tag{11.3}$$

Semileptonic D decays, $D \to K e \nu$, and neutrino and antineutrino production of charm off sea s quarks give

$$|V_{cs}| = 1.06 \pm 0.18. \tag{11.4}$$

Semileptonic B decays, $B \to D^* e \nu$, give

$$|V_{cb}| = 0.040 \pm 0.007. \tag{11.5}$$

The endpoint spectrum in semileptonic B decays, $B \to X e \nu$, give

$$|V_{ub}/V_{cb}| = 0.10 \pm 0.03. \tag{11.6}$$

(We take the various ranges for $|V_{ij}|$ above from ref. [26], except for $|V_{cb}|$ where we use an update of ref. [52].) Using unitarity constraints, one can narrow some of the above ranges (most noticeably, that of $|V_{cs}|$) and put constraints on the top mixings $|V_{ti}|$. The full information on absolute values of the CKM elements (at one sigma) from both direct measurements and unitarity is summarized by

$$|V| = \begin{pmatrix} 0.9749 - 0.9754 & 0.2187 - 0.2223 & 0.002 - 0.006 \\ 0.218 - 0.221 & 0.9735 - 0.9752 & 0.033 - 0.047 \\ 0.003 - 0.016 & 0.032 - 0.048 & 0.9986 - 0.9993 \end{pmatrix}. \tag{11.7}$$

Note that the only large uncertainties are in $|V_{ub}|$ and $|V_{td}|$. However, the two are related through eq. (10.24). Thus, the unitarity triangle is a very convenient

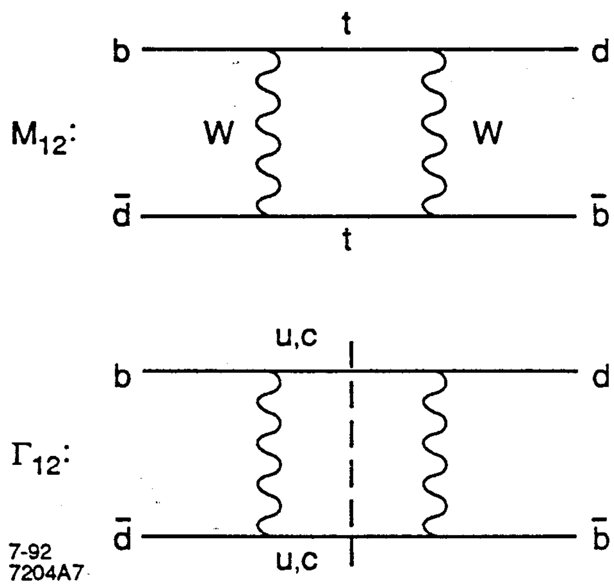

Figure 4. The quark diagram description of $M_{12}(B)$ and $\Gamma_{12}(B)$.

tool for presenting constraints from indirect measurements on the most poorly determined parameters.

The most useful CP conserving indirect measurement, namely a loop-level process, is that of mixing in the $B^0 - \bar{B}^0$ system. The experimental result is [53]

$$x_d \equiv \frac{\Delta M_B}{\Gamma_B} = 0.67 \pm 0.10. \tag{11.8}$$

The theoretical calculation is on more solid ground than in the K^0 system, because short distance physics dominate M_{12}. Thus, it can be reliably calculated from the box diagram (Fig. 4) with intermediate top quarks [54],

$$x_d = \tau_b \frac{G_F^2}{6\pi^2} \eta M_B (B_B f_B^2) m_t^2 f_2(m_t^2/m_W^2) |V_{td}^* V_{tb}|^2 \tag{11.9}$$

where

$$f_2(y) = 1 - \frac{3}{4}\frac{y(1+y)}{(1-y)^2}\left[1 + \frac{2y}{1-y^2}\ln(y)\right]. \tag{11.10}$$

Note that, typical of loop processes, there is a strong dependence on m_t which affects our ability to constrain the CKM parameters. Recently, there has been improvement in the determination of the two most uncertain parameters in (11.9) due to heavy quark symmetry considerations [52], [55, 56]:

$$\sqrt{\tau_b}|V_{cb}| = 0.040 \pm 0.05, \tag{11.11}$$

$$f_B = 190 \pm 50\ MeV. \tag{11.12}$$

The end results is that the lower bound on $|V_{td}|$ is raised to 0.006 (compare to (11.7)). However, for any specific value of m_t the information on the CKM parameters is more detailed, as presented later.

The constraints from (11.7) on the mixing angles of the standard parametrization are:

$$s_{12} = 0.2205 \pm 0.0018, \quad s_{23} = 0.040 \pm 0.007, \quad s_{13}/s_{23} = 0.10 \pm 0.03. \tag{11.13}$$

From (11.13) we find

$$J = (3.5 \pm 1.5) \times 10^{-5}\ \sin\delta. \tag{11.14}$$

12. The ϵ Parameter

As discussed in section 6.2, an approximate expression for ϵ (in a phase convention where A_2 is real) is

$$\epsilon = \frac{e^{i\pi/4}\mathrm{Im}M_{12}}{\sqrt{2}\Delta M}. \tag{12.1}$$

Furthermore, $\mathrm{Im}M_{12}$ is dominated by short distance physics and thus can be reliably calculated from the box diagrams [57]:

$$\begin{aligned} H^{box}_{\Delta S=2} =& 2\left(\frac{-ig}{\sqrt{2}}\right)^2 \sum_{i,j}(V^*_{id}V_{is})(V^*_{jd}V_{js}) \int \frac{d^4p}{(2\pi)^4}\left(\frac{-i}{p^2-m_W^2}\right)^2 \\ &\times\left(\bar{d}_{La}\gamma^\mu \frac{i(\not{p}+m_i)}{p^2-m_i^2}\gamma^\nu s^a_L\right)\left(\bar{d}_{Lb}\gamma_\nu \frac{i(\not{p}+m_j)}{p^2-m_j^2}\gamma_\mu s^b_L\right). \end{aligned} \tag{12.2}$$

There are several suppression factors in (12.2). First, it is fourth order in the weak coupling. Second, there are small mixing angles. And third, there is the GIM mechanism which guarantees that when any two up quark masses are equal, M_{12} vanishes. These three ingredients suppress M_{12} by a factor of $g^4 s_{12}^2 \frac{m_c^2}{m_W^2}$ which explains why $\Delta M_K/m_K$ is such a tiny quantity. However, there is an extra suppression factor for $\mathrm{Im}M_{12}$ from the mixing parameters:

$$\frac{\mathrm{Im}M_{12}}{\mathrm{Re}M_{12}} \sim \frac{J}{s_{12}^2} \lesssim 10^{-3}. \tag{12.3}$$

Eq. (12.3) is related to the first unitarity triangle (Fig. 2(a)): it is the ratio between its area and the length of its long basis squared or, in other words, the ratio between the height and the basis of the triangle. It is the ratio (12.3) which determines the size of ϵ in the Standard Model.

Let us make these arguments more rigorous. The phase convention independent expression for ϵ is

$$\epsilon = e^{i\pi/4}\frac{G_F^2}{12\pi^2}\frac{m_K}{\sqrt{2}\Delta M_K}\frac{(B_K f_K^2)m_W^2}{s_{12}^2}\{\eta_1 y_c \mathrm{Im}[(V_{cd}^* V_{cs} V_{ud} V_{us}^*)^2]$$
$$+\eta_2 y_t f_2(y_t)\mathrm{Im}[(V_{td}^* V_{ts} V_{ud} V_{us}^*)^2] + 2\eta_3 f_3(y_c, y_t)\mathrm{Im}[V_{cd}^* V_{cs} V_{td}^* V_{ts}(V_{ud} V_{us}^*)^2]\} \tag{12.4}$$

where $y_i = m_i^2/m_W^2$, $f_2(y)$ is given in eq. (11.10), and

$$f_3(x,y) = \ln\left(\frac{y}{x}\right) - \frac{3y}{4(1-y)}\left[1 + \frac{y}{1-y}\ln(y)\right]. \tag{12.5}$$

Well measured parameters in (12.4) are

$$\begin{aligned} G_F =& 1.166 \times 10^{-5} GeV^{-2}, \quad m_W = 80\ GeV, \\ f_K =& 0.165\ GeV, \quad \Delta M_K/m_K = 7 \times 10^{-15}. \end{aligned} \tag{12.6}$$

The factors $\eta_1 = 0.7$, $\eta_2 = 0.6$ and $\eta_3 = 0.4$ are QCD correction factors [58, 59]. The only significant uncertainty (apart, of course, from the CKM parameters which we try to determine) is in

$$B_K = 2/3 \pm 1/3. \tag{12.7}$$

We can write (12.4) in a way which makes the dependence on J manifest:

$$\epsilon = 4 \times 10^4 e^{i\pi/4} B_K J\{[\eta_3 f_3(y_c, y_t) - \eta_1]y_c + \eta_2 y_t f_2(y_t)\mathrm{Re}(V_{td}^* V_{ts} V_{ud} V_{us}^*)/s_{12}^2\}. \tag{12.8}$$

The terms in curly brackets are $\mathcal{O}(10^{-3})$. If $|\epsilon|$ were much larger than $\mathcal{O}(10^{-3})$, it would have contradicted the Standard Model explanation of CP violation as arising from the single phase in the CKM matrix. But as long as $|\epsilon| \lesssim \mathcal{O}(10^{-3})$ it does not really test the CKM picture but merely fixes the value of $\sin\delta$. Yet, the fact that the experimental value is

$$|\epsilon| = (2.258 \pm 0.018) \times 10^{-3}, \tag{12.9}$$

implying $\sin\delta \sim \mathcal{O}(1)$ and not much smaller, makes the CKM picture phenomenologically attractive: CP violation as observed in the neutral kaon system is conveniently accommodated in the Standard Model.

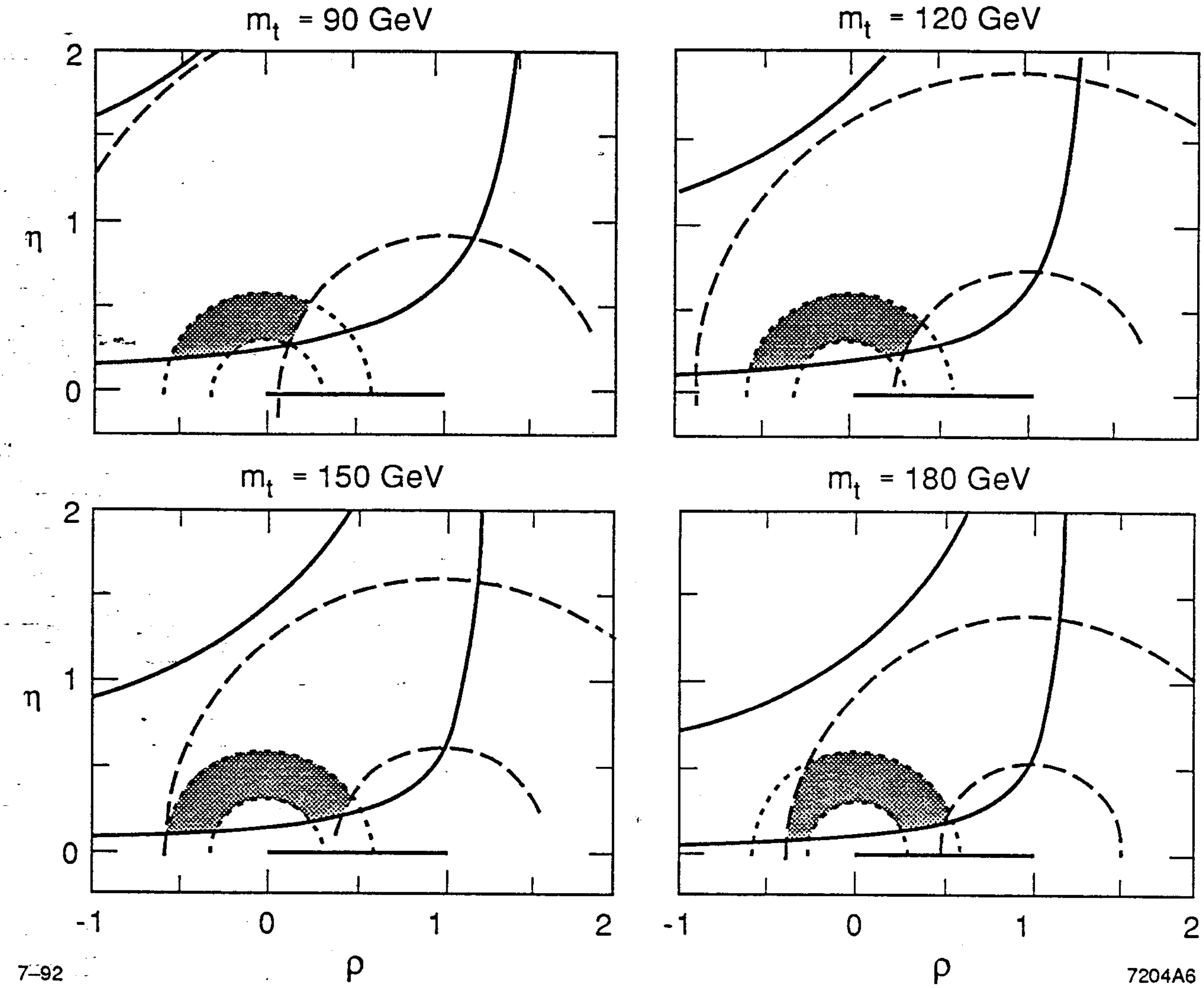

Figure 5. Constraints on the unitarity triangle from ϵ (solid curves), x_d (dashed curves) and $|V_{ub}/V_{cb}|$ (dotted curves) for various top masses. The dotted area gives the final allowed range.

The detailed constraints on the Standard Model parameters are presented in Fig. 5. The ϵ constraint (12.4) requires that the vertex A of the unitarity triangle lies between two hyperbolae. The width of the allowed band is determined mainly by the uncertainty in B_K. The bounds on the mixing parameters depend on the yet unknown mass of the top, so we give the constraints for various top masses within the experimentally allowed range, $91 \leq m_t \leq 180$ GeV. Also presented are the direct measurement of $|V_{ub}/V_{cb}|$ (eq. (11.6)), and the indirect measurement of $|V_{td}V_{tb}|$ from $B - \bar{B}$ mixing (eq. (11.9)). The phase δ of the standard parametrization is the same as the angle γ of the unitarity triangle. We see that indeed all constraints can be met consistently in the Standard Model; the measurement of ϵ is the one which requires $\delta \neq 0$, or more explicitly

$$20^o \leq \delta \lesssim 178^o. \qquad (12.10).$$

Note that sign[Re(ϵ)] reveals that J is positive or, equivalently, that δ is in one of the first two quadrants. For the rescaled unitarity triangle, this means that the vertex A lies in the upper half plane or, equivalently, that η is positive.

13. The ϵ'/ϵ Parameter

The most recent measurements of ϵ'/ϵ give [36, 37]

$$\mathrm{Re}(\epsilon'/\epsilon) = \begin{cases} (2.3 \pm 0.7) \times 10^{-3} & \text{NA31}, \\ (0.6 \pm 0.7) \times 10^{-3} & \text{E731}. \end{cases} \qquad (13.1)$$

Thus, there is yet no compelling evidence for direct CP violation: while consistent with the Standard Model predictions, the weighted average for ϵ'/ϵ is only two standard deviations from zero.

The calculation of ϵ'/ϵ has many theoretical uncertainties. Let us first isolate the important ingredients in the calculation and try to get an order of magnitude estimate. There are several types of diagrams that contribute to $K \to \pi\pi$. First, there are tree diagrams of both exchange and spectator types. The exchange diagram contributes only to the final $I = 0$ state, while the spectator diagrams contribute to both $I = 0$ and $I = 2$ final states. All tree diagrams have a common weak phase,

$$\phi_T = \arg(V_{ud}^* V_{us}). \tag{13.2}$$

Second, there are three penguin diagrams, one for each intermediate charge 2/3 quark. They all contribute to the final $I = 0$ state only. However, each depends on a different CKM combination:

$$\phi_P^q = \arg(V_{qd}^* V_{qs}). \tag{13.3}$$

A difference in the weak phases between A_0 and A_2 is then a result of the fact that A_0 has contributions from penguin diagrams with intermediate c and t quarks. Consequently, ϵ' is suppressed by the following factors:

a. $|A_2/A_0| \sim 0.045$.

b. $|A_0^{\text{penguin}}/A_0^{\text{tree}}| \sim 0.05$. (There is no relative weak phase between the tree contributions to A_0 and to A_2.)

c. $|(V_{td}^* V_{ts})/(V_{cd}^* V_{cs})| \sim 10^{-3}$. (The penguin diagrams with intermediate u or c quarks give a contribution which is dominated by the same weak phase as the tree diagrams.)

The last factor is $\mathcal{O}(J/s_{12}^2) \sim \epsilon$. Thus, it cancels in the ratio ϵ'/ϵ, and we are left with (the very rough) order of magnitude estimate, $\epsilon'/\epsilon \sim 10^{-3}$.

The actual calculation is very complicated. It can be cast into the form (see ref. [60] and references therein)

$$\epsilon'/\epsilon \approx 300\ J\ \frac{G_F}{\sqrt{2}} \frac{s_{12}}{\text{Re}A_0} y_6 \langle Q_6 \rangle [1 - \Omega_{\eta+\eta'} - \Omega_{EWP} - (\Omega_8 + \Omega_{27} + \Omega_P)]. \tag{13.4}$$

The y_6 factor is the Wilson coefficient for the operator

$$Q_6 = -8 \sum_{q=u,d,s} (\bar{s}_L q_R)(\bar{q}_R d_L) \tag{13.5}$$

which describes the strong penguin contribution. The matrix element of Q_6,

$$\langle Q_6 \rangle = -1.16 \; GeV^3 \left[\frac{175 \; MeV}{m_s(\mu)} \right] \frac{(m_K^2 - m_\pi^2)}{\Lambda_\chi^2}, \tag{13.6}$$

(given here in the $1/N$ approach) is very sensitive to the mass of the strange quark, and introduces large uncertainties into the calculation. The various Ω's give the relative contribution of other four quark operators. The three Ω's in parenthesis are small ($\lesssim 0.2$ in absolute value). $\Omega_{\eta+\eta'}$ represents isospin breaking effects in quark masses and does not depend on m_t,

$$\Omega_{\eta+\eta'} \approx 0.3. \tag{13.7}$$

However, the contribution from electroweak penguins is rather large and depends sensitively on m_t. For $\Lambda_{QCD} = 100$ MeV and $m_s = 200$ MeV, ref. [60] quotes

$$\Omega_{EWP} \sim \begin{cases} -0.04 & m_t = 100 \; GeV, \\ +0.21 & m_t = 150 \; GeV, \\ +0.56 & m_t = 200 \; GeV. \end{cases} \tag{13.8}$$

Thus, for large m_t (~ 200 GeV) there is a cancellation among the various contributions in (13.4), providing yet another, somewhat accidental, suppression factor for ϵ'/ϵ. It leads to the conclusion that the Standard Model would not be excluded if $\epsilon'/\epsilon \sim 0$. The calculation is consequently even more sensitive to hadronic uncertainties.

To summarize, ϵ'/ϵ gets contributions from many four quark operators. The hadronic matrix elements of these operators involve large uncertainties. The fact that various operators contribute with similar order of magnitude but with differing signs, enhances the uncertainties. The result is very sensitive to the top mass. A wide range of ϵ'/ϵ values can be accommodated in the Standard Model. Ref. [60] finds, for example,

$$\begin{aligned} 2 \times 10^{-4} \lesssim \epsilon'/\epsilon \lesssim 3 \times 10^{-3} \quad & m_t = 100 \; GeV, \\ 3 \times 10^{-5} \lesssim \epsilon'/\epsilon \lesssim 2 \times 10^{-4} \quad & m_t = 200 \; GeV. \end{aligned} \tag{13.9}$$

14. *CP* Asymmetries in Neutral *B* Decays

14.1. MEASURING THE ANGLES OF THE UNITARITY TRIANGLES

As mentioned in section 3.2, in the B system we expect model independently that $\Gamma_{12} \ll M_{12}$. However, within the Standard Model and assuming that the box diagram (with a cut) is appropriate to estimate Γ_{12}, we can actually calculate the two quantities from the quark diagrams in Fig. 4. The calculation gives (see ref. [15] and references therein)

$$\frac{\Gamma_{12}}{M_{12}} = \frac{3\pi}{2}\frac{1}{f_2(y_t)}\frac{m_b^2}{m_t^2}\left(1 + \frac{8}{3}\frac{m_c^2}{m_b^2}\frac{V_{cb}V_{cd}^*}{V_{tb}V_{td}^*}\right). \tag{14.1}$$

This confirms our order of magnitude estimate, $|\Gamma_{12}/M_{12}| \lesssim 10^{-2}$. Thus, to a very good approximation,

$$\left(\frac{q}{p}\right)_B = \sqrt{\frac{M_{12}^*}{M_{12}}} = \frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}. \tag{14.2}$$

We will use this result in our calculations of *CP* violation in the interference of mixing and decay. However, before doing that we note that (14.1) allows an estimate of *CP* violation in mixing, namely

$$\left|\frac{q}{p}\right| - 1 = \frac{1}{2}\mathrm{Im}\frac{\Gamma_{12}}{M_{12}} = \frac{4\pi}{f_2(y_t)}\frac{m_c^2}{m_t^2}\frac{J}{|V_{tb}V_{td}^*|^2} \sim 10^{-3}. \tag{14.3}$$

Notice that the last term is the ratio of the area of the unitarity triangle to the length of one of its sides squared, so it is $\mathcal{O}(1)$. (For the B_s system, $J/|V_{tb}V_{ts}^*|^2 \sim 10^{-2}$, as can be seen from the unitarity triangles in Fig. 2.) The only suppression factor is then (m_c^2/m_t^2). The uncertainty in the calculation comes from the use of a quark diagram to describe Γ_{12}, and could be a factor of 2–3.

Now we turn back to decays into *CP* eigenstates. We would like to choose modes dominated by a single diagram because these, as explained above, are theoretically clean. However, most channels have contributions from both tree and penguin diagrams. The ratio between the two for a decay $b \to q\bar{q}'q'$ is [61, 62, 63]

$$\frac{\text{penguin}}{\text{tree}} \approx \left(\frac{\alpha_s}{12\pi} \ln \frac{m_t^2}{m_b^2}\right) \frac{V_{tb}V_{tq}^*}{V_{q'b}V_{q'q}^*} \frac{\langle\text{penguin}\rangle}{\langle\text{tree}\rangle}. \tag{14.4}$$

The factor in parenthesis is $\mathcal{O}(0.02)$, but it may be partially compensated by the ratio of matrix elements. Thus, there are three appropriate classes:

(*i*) Modes with $\left|\frac{V_{tb}V_{tq}^*}{V_{q'b}V_{q'q}^*}\right| \lesssim 1$. Examples are $B \to \pi\pi$, $B \to DD$, $B_s \to \rho K_S$ and $B_s \to \psi K_S$.

(*ii*) Modes with no tree contribution. Examples are $B \to \phi K_S$, $B \to K_S K_S$, $B_s \to \eta'\eta'$ and $B_s \to \phi K_S$.

(*iii*) Modes with $\arg\left(\frac{V_{tb}V_{tq}^*}{V_{q'b}V_{q'q}^*}\right) = 0,\ \pi$. Examples are $B \to \psi K_S$ and $B_s \to \phi\psi$.

Our first example is $B \to \pi\pi$. The quark subprocess is $b \to u\bar{u}d$ which is dominated by a W-mediated tree diagram. Thus, to a good approximation

$$\frac{\bar{A}_{\pi\pi}}{A_{\pi\pi}} = \frac{V_{ub}V_{ud}^*}{V_{ub}^*V_{ud}}. \tag{14.5}$$

Combining (14.2) and (14.5), we find

$$\lambda(B \to \pi^+\pi^-) = \left(\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\right)\left(\frac{V_{ud}^*V_{ub}}{V_{ud}V_{ub}^*}\right) \Longrightarrow \text{Im}\lambda_{\pi\pi} = \sin(2\alpha). \tag{14.6}$$

The penguin contribution to this decay has a weak phase, $\arg(V_{td}^*V_{tb})$, different from the tree diagram, so it may modify both $|\lambda|$ and $\text{Im}\lambda$. We estimate that the resulting hadronic uncertainty is $\lesssim 0.1$, but it can be eliminated using isospin analysis [31, 32, 33].

The analysis of $B \to D^+D^-$ proceeds along very similar lines. The quark subprocess here is $b \to c\bar{c}d$, and so

$$\lambda(B \to D^+D^-) = \left(\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\right)\left(\frac{V_{cd}^*V_{cb}}{V_{cd}V_{cb}^*}\right) \Longrightarrow \text{Im}\lambda_{DD} = -\sin(2\beta). \tag{14.7}$$

Again, there may be a small hadronic uncertainty due to penguin contributions.

The same weak phase can be measured without hadronic uncertainties in $B \to \psi K_S$. A new ingredient in the analysis is the effect of $K - \bar{K}$ mixing. For decays with a single K_S in the final state, $K - \bar{K}$ mixing is essential because $B^0 \to K^0$ and $\bar{B}^0 \to \bar{K}^0$, and interference is possible only due to $K - \bar{K}$ mixing. This adds a factor of

$$\left(\frac{q}{p}\right)_K = \frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}} \qquad (14.8)$$

into $(\bar{A}/A)$. The quark subprocess in $\bar{B}^0 \to \psi\bar{K}^0$ is $b \to c\bar{c}s$ which is, again, dominated by a W-mediated tree diagram:

$$\frac{\bar{A}_{\psi K_S}}{A_{\psi K_S}} = -\left(\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}\right)\left(\frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}\right). \qquad (14.9)$$

The minus sign on the right hand side of (14.9) is a result of ψK_S being a CP odd state. Combining (14.2) and (14.9), we get

$$\lambda(B \to \psi K_S) = -\left(\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\right)\left(\frac{V_{cd}^*V_{cb}}{V_{cd}V_{cb}^*}\right) \Longrightarrow \mathrm{Im}\lambda_{\psi K_S} = \sin(2\beta). \qquad (14.10)$$

The theoretical advantage of using this mode is the following. As in previous cases, there is a small penguin contribution to the direct decay in this process as well. However, its weak phase, $\arg(V_{tb}V_{ts}^*)$, is similar (mod π) to the weak phase of the tree decay and thus affects neither $|\lambda|$ nor $\mathrm{Im}\lambda$. Thus, eq. (14.10) is clean of hadronic uncertainties to $\mathcal{O}(10^{-3})$ – *This gives the theoretically cleanest determination of a CKM parameter, even cleaner than the determination of* $\sin\theta_C$ *from* $K \to \pi\ell\nu$.

The third angle of the unitarity triangle (γ) can be measured in B_s decays.★ Calculations in the B_s system are very similar to the B^0 system. One finds,

★ This method for measuring γ seems to be experimentally very difficult. Various alternative ways were suggested [64, 65, 66].

similar to (14.2),

$$\left(\frac{q}{p}\right)_{B_s} = \sqrt{\frac{M_{12}^*}{M_{12}}} = \frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*}. \tag{14.11}$$

It is then straightforward to show that

$$\lambda(B_s \to \rho K_S) = \left(\frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*}\right) \left(\frac{V_{ud}^* V_{ub}}{V_{ud} V_{ub}^*}\right) \left(\frac{V_{cs}^* V_{cd}}{V_{cs} V_{cd}^*}\right) \Longrightarrow \mathrm{Im}\lambda_{\rho K_S} \approx -\sin(2\gamma). \tag{14.12}$$

In the last equation we neglected a small correction of $\mathcal{O}(\beta')$, where β' is an angle in the unitarity triangle (10.23):

$$\beta' \equiv \arg\left[-\frac{V_{cs} V_{cb}^*}{V_{ts} V_{tb}^*}\right]. \tag{14.13}$$

Another interesting possibility is the study of tree-forbidden B decays, for example $B \to \phi K_S$. The quark subprocess $b \to s\bar{s}s$ involves flavor changing neutral current and cannot proceed via a tree level Standard Model diagram. The leading contribution comes then from penguin diagrams. In general (as is the case in K decays), each of the three penguin diagrams is of different magnitude and phases, inducing direct CP violation. But here, to a very good approximation, the diagrams with intermediate u and c quarks are of similar magnitude except for their CKM factors and their strong phases are very small. Using unitarity one finds that

$$\frac{\bar{A}_{\phi K_S}}{A_{\phi K_S}} = \left(\frac{V_{ts}^* V_{tb}}{V_{ts} V_{td}^*}\right) \left(\frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*}\right), \tag{14.14}$$

which leads to (neglecting $\mathcal{O}(\beta')$ corrections)

$$\mathrm{Im}\lambda_{\phi K_S} = -\sin(2\beta). \tag{14.15}$$

Our final example is $B_s \to D_s^+ D_s^-$. The quark subprocess is $b \to c\bar{c}s$, so that

$$\lambda(B_s \to D_s^+ D_s^-) = \left(\frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*}\right) \left(\frac{V_{cs}^* V_{cb}}{V_{cs} V_{cb}^*}\right) \Longrightarrow \mathrm{Im}\lambda_{D_s D_s} = -\sin(2\beta'). \tag{14.16}$$

There are five quark subprocesses in each of B^0 and B_s decays which are expected to be dominated by a single CKM phase, so that the leading *CP* violating effect is interference between mixing and decay. We list them in tables 1 and 2. The list of hadronic final states gives examples only. Other states may be more favorable experimentally. We always quote the *CP* asymmetry for *CP* even states, regardless of the specific hadronic state listed. In previous analyses in the literature, the approximation $\beta' = 0$ is used.

TABLE 1

CP Asymmetries in B Decays

Final state	Quark sub-process	SM prediction
ψK_S	$\bar{b} \to \bar{c}c\bar{s}$	$-\sin 2\beta$
D^+D^-	$\bar{b} \to \bar{c}c\bar{d}$	$-\sin 2\beta$
$\pi^+\pi^-$	$\bar{b} \to \bar{u}u\bar{d}$	$\sin 2\alpha$
ϕK_S	$\bar{b} \to \bar{s}s\bar{s}$	$-\sin 2(\beta - \beta')$
$K_S K_S$	$\bar{b} \to \bar{s}s\bar{d}$	0

TABLE 2

CP Asymmetries in B_s Decays

Final state	Quark sub-process	SM prediction
$D_s^+ D_s^-$	$\bar{b} \to \bar{c}c\bar{s}$	$-\sin 2\beta'$
ψK_S	$\bar{b} \to \bar{c}c\bar{d}$	$-\sin 2\beta'$
ρK_S	$\bar{b} \to \bar{u}u\bar{d}$	$-\sin 2(\gamma + \beta')$
$\eta'\eta'$	$\bar{b} \to \bar{s}s\bar{s}$	0
ϕK_S	$\bar{b} \to \bar{s}s\bar{d}$	$\sin 2(\beta - \beta')$

The allowed ranges for the angles α, β, γ and β' are found from the various constraints on the form of the unitarity triangles [67, 68, 69, 70, 56]. The simplest to study is β'. Note that β' is the angle in the triangle related to (11.6) and therefore it is very small. Explicitly

$$|\sin 2\beta'| = 2|(\sin\gamma)V_{us}V_{ub}/(V_{cs}V_{cb})| \leq 0.06. \tag{14.17}$$

The bound is saturated when $\sin\gamma = 1$ and $|V_{ub}/V_{cb}| = 0.13$. However, from the lower bounds on these quantities, we find that $\sin 2\beta'$ could be as small as 10^{-3}, in which case the hadronic uncertainties, which we neglected, become important.

Experimentally, CP asymmetries in B decays are likely to be measured long before those in B_s decays. Thus, we now concentrate in the Standard Model predictions for $\sin 2\alpha$ and $\sin 2\beta$. We will present our results directly in the $\sin 2\alpha - \sin 2\beta$ plane [71]. These are the quantities measured (see Table 1) and, furthermore, it allows a direct comparison of the Standard Model predictions with models of new physics where the asymmetries are not necessarily related to angles of the unitarity triangle [72].

We use the following relations to transform from the (ρ, η) coordinates of the free vertex A of the unitarity triangle to $(\sin 2\alpha, \sin 2\beta)$:

$$\begin{aligned} \sin 2\alpha &= \frac{2\eta[\eta^2 + \rho(\rho - 1)]}{[\eta^2 + (1-\rho)^2][\eta^2 + \rho^2]}, \\ \sin 2\beta &= \frac{2\eta(1-\rho)}{\eta^2 + (1-\rho)^2}. \end{aligned} \tag{14.18}$$

Note that these coordinate transformations are highly nonlinear; hence the predictions in the $\sin 2\alpha - \sin 2\beta$ plane will be very different from the more familiar constraints in the $\rho - \eta$ plane. Furthermore, since (14.18) are not invertible, we may not simply map the regions in the $\rho - \eta$ plane allowed by each of the various constraints into corresponding regions in the $\sin 2\alpha - \sin 2\beta$ plane, and then assume that the overlap in the latter is allowed. To see this, note that a single point in the overlap region in the $\sin 2\alpha - \sin 2\beta$ plane may correspond to two different points in the $\rho - \eta$ plane. If each of these two points is allowed by one constraint but forbidden by the other, then the original point in the $\sin 2\alpha - \sin 2\beta$ plane is in fact forbidden though it is in the overlap of two regions allowed by the individual constraints. We therefore form the overlap in the $\rho - \eta$ plane first, and then map this overall-allowed region into $\sin 2\alpha - \sin 2\beta$ coordinates.

Since the x_d and ϵ constraints depend on m_t, we have carried out our analysis for various m_t values within the range 90 GeV$\leq m_t \leq$ 185 GeV. We present our analysis in Fig. 7 in two ways [71]. First, the solid curves encompass all values of $(\sin 2\alpha, \sin 2\beta)$ which satisfy all three constraints using values of the input parameters within their $1 - \sigma$ ranges (or within the theoretically favored ranges for the parameters B_K and f_B). That is, the SM can accommodate a B-factory result anywhere within these curves without stretching any input parameter beyond its $1 - \sigma$ range. We will refer to these regions as the "allowed" areas of the SM.

Second, in order to get a sense of the expected value of $(\sin 2\alpha, \sin 2\beta)$ given our current knowledge of the various input parameters, we generated numerous sample values for these parameters based on a Gaussian distribution for $|V_{cd}|$, $\tau_B |V_{cb}|^2$, $|V_{ub}/V_{cb}|$, τ_B, x_d, m_c and $|\epsilon|$, and a uniform distribution (= 0 outside of the "$1 - \sigma$" range) for f_B. For each sample set we used the constraints (11.6) and (11.9) to determine ρ and η, and then rejected those sets which did not meet the constraint (12.4) for $1/3 \leq B_K \leq 1$. We binned the sets which passed in the $\sin 2\alpha - \sin 2\beta$ plane, and thus obtained their probability distribution. We show in fig. 6 the resulting 90% probability contours in dashed curves. Since we do not know the true origin of the CKM parameters and thus do not know the true probability distribution from which the experimental inputs result, and since the theoretical restrictions on f_B and B_K cannot be posed statistically, we can only interpret these probability contours as an indication of likely outcomes for B-factory results based on the SM. For example, the "tail" of the allowed

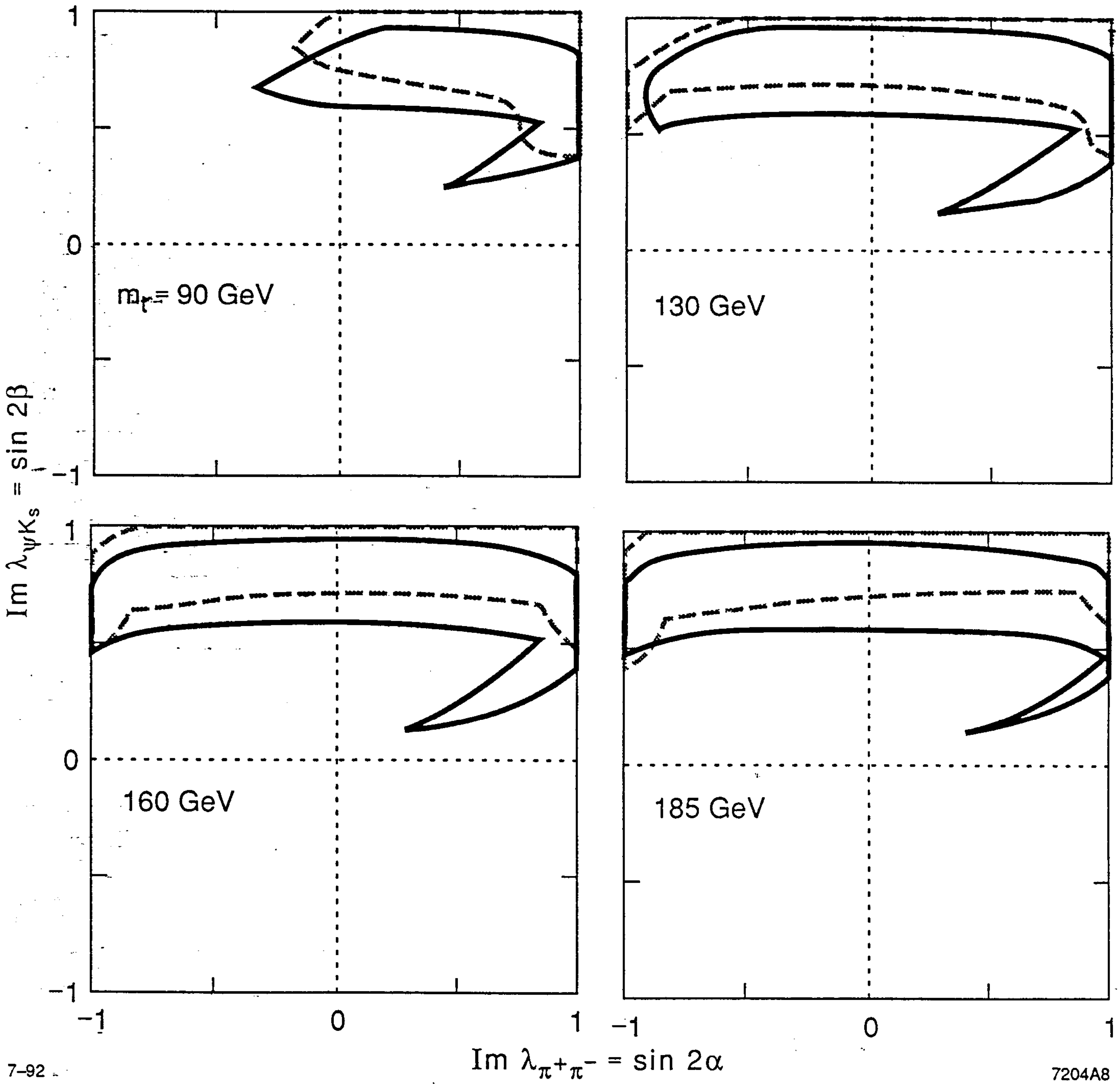

Figure 6. The Standard Model allowed range for the asymmetries in $B \to \psi K_S$ and $B \to \pi^+\pi^-$ (solid curves). The 90% C.I. range is given in dashed curves (see text).

areas which extends towards small values of $(\sin 2\alpha, \sin 2\beta)$ requires many of the parameters to be stretched to their $1-\sigma$ bounds and so seems unlikely and lies outside the probability contour.

We find that $\sin 2\alpha$ can have any value in the full range from -1 to 1, while $\sin 2\beta$ is always positive and has a lower bound [71]

$$\sin 2\beta \geq 0.15\,. \tag{14.19}$$

Note that none of the angles is allowed to vanish due to the ϵ constraint. The fact that $\sin 2\phi$ may vanish for a certain angle is actually a result of the possibility $\phi = \pi/2$. However, due to $|V_{ub}/V_{cb}| < |V_{cd}|$, $\beta < \pi/2$ (actually, $\beta \lesssim \pi/5$) and hence the lower bound in (14.19).

We further find that $\sin 2\alpha$ is likely to be positive if the top mass is near its present lower bound, and most importantly *the favored values for* $\sin 2\beta$ *are above 0.5.* We also find that the bounds on the two quantities are correlated. In particular, we note that:

a. The magnitude of at least one of the two asymmetries is always larger than 0.2, and probably larger than 0.6.

b. If $\sin 2\beta \leq 0.4$, then $\sin 2\alpha$ must be positive—in fact, above 0.2.

Once the top mass is measured firmer predictions will of course be possible, based on one of the graphs in fig. 6.

We conclude that neutral B mesons provide many decay modes into final CP eigenstates which have CP violation purely from interference between mixing and decay. The asymmetries are expected to be large, and the hadronic uncertainties enter only at $\mathcal{O}(10^{-3})$.

15. The EDM of the Neutron

The Standard Model prediction for the EDM of the neutron is extremely small. First, we discuss the contributions from quark EDMs. One loop diagrams do not contribute. The reason is that any one loop diagrams that contributes to $\mathcal{D}_q$ $(q = d, u)$ is proportional to $V_{iq}^* V_{iq}$; the phase cancels out and no CP violating effects are possible. Two loop diagrams do not contribute either [73]. Here there is no intuitive reason. Actually, individual two loop diagrams do not vanish. But an explicit calculation shows that the sum of all two loop diagrams vanishes. Consequently, the leading contribution to $\mathcal{D}_n$ comes from three loop diagrams. There is no explicit calculation available, but only an order of magnitude estimate:

$$\mathcal{D}_n \sim e m_q \frac{G_F \alpha \alpha_s}{\pi^4} \frac{m_t^2 m_s^2}{m_W^4} J \leq 10^{-33} \; e \text{ cm}. \tag{15.1}$$

The calculation of diagrams other than $\mathcal{D}_q$ is subject to even larger uncertainties. It seems unlikely, however, that Standard Model mechanisms give $\mathcal{D}_n$ larger by more than three orders of magnitudes than (15.1). It seems then that, if the KM phase is the only source of CP violation, the EDM of the neutron is much too small to be experimentally observed in the foreseeable future. (This feature makes it a very sensitive probe of physics beyond the Standard Model!)

As mentioned in our introductory discussion of the EDM of the neutron, the QCD lagrangian will generally include a CP violating term of the form

$$\mathcal{L}_\theta = \frac{g_s^2}{32\pi^2} \theta G_a^{\mu\nu} \tilde{G}_{a\mu\nu}. \tag{15.2}$$

We found that the upper bound on $\mathcal{D}_n$ requires that θ is extremely small, $\theta \lesssim 10^{-9}$. The important point about the Standard Model of electroweak interactions in this regard is that it makes it impossible to avoid this problem by requiring CP symmetry so that there is no term of the form (15.2). The reason is that in the Standard Model, CP is explicitly broken. The actual parameters which contributes to $\mathcal{D}_n$ is not θ but rather the combination

$$\bar{\theta} = \theta + \arg[\det M]. \tag{15.3}$$

Thus, without extending the Standard Model, there is no natural way to suppress the effects of $\bar{\theta}$.

16. Summary

The Standard Model predicts that all CP violating phenomena in neutral meson decays are related to the single phase of the Cabibbo–Kobayashi–Maskawa matrix. Consequently, the model is very predictive. CP violation as observed in the K system (the ϵ parameter) is conveniently accommodated in the Standard Model. Together with other (CP conserving) measurements of CKM parameters it gives clean predictions for large CP asymmetries in neutral B decays. Their mesurement in the future will stringently test the CKM picture of CP violation. The KM phase gives tiny electric dipole moments for the neutron and the electron. If either of them is found in near future experiments, it will unambiguously require a source of CP violation additional to δ_{KM}. On the other hand, the smallness of $\mathcal{D}_n$ requires extreme fine tuning of θ_{QCD} and implies that our understanding of CP violation is incomplete.

III. *CP* VIOLATION BEYOND THE STANDARD MODEL

17. Extending the Quark Sector

Z-Mediated FCNC

17.1. INTRODUCTION

In this chapter, we update the analysis of refs. [74, 75]. We study a model with an extended quark sector. In addition to the three standard generations of quarks, there is an $SU(2)_L$-singlet of charge $-1/3$. For our purposes, the important feature of this model is that it allows for CP violating Z-mediated Flavor Changing Neutral Currents (FCNC).

To understand how these FCNC arise, it is convenient to work in the basis where the up sector interaction eigenstates are identified with the mass eigenstates. The down sector interaction eigenstates are then related to the mass eigenstates by a 4×4 unitary matrix K. Charged current interactions are described by

$$\begin{aligned} \mathcal{L}^W_{int} =& \frac{g}{\sqrt{2}}(W^-_\mu J^{\mu^+} + W^+_\mu J^{\mu^-}), \\ J^{\mu-} =& V_{ij}\bar{u}_{iL}\gamma^\mu d_{jL}. \end{aligned} \tag{17.1}$$

The charged current mixing matrix V is a 3×4 sub-matrix of K:

$$V_{ij} = K_{ij} \quad \text{for} \quad i = 1, \ldots, 3; \; j = 1, \ldots, 4. \tag{17.2}$$

Note that V is parametrized by six real angles and *three* phases, instead of three angles and one phase in the original CKM matrix. As we shall see, all three phases may affect CP asymmetries in B^0 decays.

Neutral current interactions are described by

$$\begin{aligned}\mathcal{L}^Z_{int} =& \frac{g}{\cos\theta_W} Z_\mu (J^{\mu 3} - \sin^2\theta_W J^\mu_{EM}),\\ J^{\mu 3} =& -\tfrac{1}{2} U_{pq} \bar{d}_{pL}\gamma^\mu d_{qL} + \tfrac{1}{2}\delta_{ij}\bar{u}_{iL}\gamma^\mu u_{jL}.\end{aligned} \tag{17.3}$$

The neutral current mixing matrix for the down sector is $U = V^\dagger V$. As V is not unitary, $U \neq \mathbf{1}$. In particular, its non-diagonal elements do not vanish:

$$U_{pq} = -K^*_{4p} K_{4q} \quad \text{for} \quad p \neq q. \tag{17.4}$$

The three elements which are relevant for our study are

$$\begin{aligned}U_{ds} =& V^*_{ud}V_{us} + V^*_{cd}V_{cs} + V^*_{td}V_{ts},\\ U_{db} =& V^*_{ud}V_{ub} + V^*_{cd}V_{cb} + V^*_{td}V_{tb},\\ U_{sb} =& V^*_{us}V_{ub} + V^*_{cs}V_{cb} + V^*_{ts}V_{tb}.\end{aligned} \tag{17.5}$$

The fact that, unlike the SM, the various U_{pq} do not necessarily vanish, allows FCNC at tree level. This may substantially modify the analysis of CP asymmetries.

17.2. EXPERIMENTAL CONSTRAINTS ON THE U_{pq} ELEMENTS

The flavor changing couplings of the Z contribute to various FCNC processes:

(*i*) ΔM_K, the mass difference between the neutral kaons.

$$(\Delta M_K)_Z = \frac{\sqrt{2} G_F B_K f_K^2 M_K \eta_1}{6} \left|\mathrm{Re}[(U_{ds})^2]\right|. \tag{17.6}$$

(*ii*) ϵ, the CP violating parameter in the K system.

$$|\epsilon|_Z = \frac{G_F B_K f_K^2 M_K \eta_1}{12 \Delta M_K} \left|\mathrm{Im}[(U_{ds})^2]\right|. \tag{17.7}$$

(*iii*) $K_L \to \mu^+\mu^-$.

$$\frac{\tau(K^+)BR(K_L \to \mu^+\mu^-)_Z}{\tau(K_L)BR(K^+ \to \mu^+\nu)} = 2\left[\left(\tfrac{1}{2} - \sin^2\theta_W\right)^2 + \left(\sin^2\theta_W\right)^2\right] \frac{(\mathrm{Re}\, U_{ds})^2}{|V_{us}|^2}. \tag{17.8}$$

(*iv*) $B \to \ell^+\ell^- X$.

$$\frac{BR(B \to \ell^+\ell^- X)_Z}{BR(B \to \ell\nu X)} = \left[\left(\tfrac{1}{2} - \sin^2\theta_W\right)^2 + \left(\sin^2\theta_W\right)^2\right] \frac{|U_{db}|^2 + |U_{sb}|^2}{|V_{ub}|^2 + F_{ps}|V_{cb}|^2}. \tag{17.9}$$

(*v*) x_d, the mixing parameter in the B system.

$$(x_d)_Z = \frac{\sqrt{2}G_F B_B f_B^2 m_B \eta \tau_b}{6}|U_{db}|^2. \tag{17.10}$$

The experimental measurements of these processes puts severe constraints [76, 77, 74, 75] on the flavor changing couplings of the Z boson (U_{pq}):

$$|\mathrm{Re}\, U_{ds}| \leq 2.4\times10^{-5}, \quad |\mathrm{Im}\, U_{ds}| \leq \min\{6.4\times10^{-4},\ 1.3\times10^{-9}/|\mathrm{Re}\, U_{ds}|\}, \tag{17.11}$$

$$|U_{db}/V_{cb}| \leq 0.037, \quad |U_{sb}/V_{cb}| \leq 0.041. \tag{17.12}$$

17.3. IMPLICATIONS OF Z-MEDIATED FCNC

If the U_{pq} elements are not negligibly small, they will affect many aspects of physics related to CP asymmetries in B decays:

(*i*) Mixing of neutral mesons.

The experimentally measured values of mixing in the K and B systems can be explained by SM processes. Still, the uncertainties in the theoretical calculations (such as in the values of B_K, f_B or V_{td}) allow a situation where SM processes do not give the dominant contributions to various mixing processes. For example,

$$(x_d)_{\rm box} = 0.024\ y_t f_2(y_t) \left[\frac{\sqrt{B_B} f_B}{0.14\ GeV}\right]^2 \left[\frac{\tau_b |V_{cb}|^2}{2.3\times 10^9\ GeV^{-1}}\right] \left[\frac{|V_{td}/V_{cb}|}{0.09}\right]^2, \tag{17.13}$$

namely, the Standard Model box diagram could contribute as little as 3% of the experimental value of x_d, and even less if unitarity of the CKM matrix does not hold, in which case the lower bound $|V_{td}/V_{cb}| \geq 0.09$ can be violated. Instead, it is possible that the dominant mechanism is Z-mediated FCNC. We will now find how large should the elements of the neutral current mixing matrix be in order that this would be the case.

For $K - \bar{K}$ mixing to be dominated by Z-mediated tree level diagrams, eq. (17.6) requires

$$|{\rm Re}[(U_{ds})^2]| \geq 1.4 \times 10^{-7}. \tag{17.14}$$

For ϵ to be dominated by Z-mediated tree level diagrams, eq. (17.7) requires

$$|{\rm Im}[(U_{ds})^2]| \geq 0.9 \times 10^{-9}. \tag{17.15}$$

For $B_q - \bar{B}_q$ mixing to be dominated by Z-mediated tree level diagrams, eq. (17.10) requires

$$|U_{db}/V_{cb}| \geq 0.014; \quad |U_{db}/(V_{td}^* V_{tb})| \geq 0.08; \quad |U_{sb}/(V_{ts}^* V_{tb})| \geq 0.08. \tag{17.16}$$

Note that if unitarity is only weakly violated, so that $|V_{ts}V_{tb}| \sim |V_{cs}V_{cb}|$, then the last requirement in (17.16) is in contradiction with (17.12) and cannot be fulfilled, implying that the dominant mechanism for B_s mixing is still the Standard Model box diagram.

(*ii*) Unitarity of the 3×3 CKM matrix.

Within the SM, unitarity of the three generation CKM matrix gives:

$$\begin{aligned}\mathcal{U}_{ds} &\equiv V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts} = 0,\\ \mathcal{U}_{db} &\equiv V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0,\\ \mathcal{U}_{sb} &\equiv V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} = 0.\end{aligned} \tag{17.17}$$

However, eq. (17.5) shows that now eq. (17.17) is replaced by

$$\mathcal{U}_{ds} = U_{ds}; \quad \mathcal{U}_{db} = U_{db}; \quad \mathcal{U}_{sb} = U_{sb}. \tag{17.18}$$

A measure of the violation of (17.17) is given by

$$|U_{ds}|/|V_{ud}^* V_{us}| \lesssim 10^{-4}; \quad |U_{sb}|/|V_{cs}^* V_{cb}| \leq 0.04; \quad |U_{db}|/|V_{cd}^* V_{cb}| \leq 0.17. \tag{17.19}$$

These bounds follow from the experimental bounds given above. The first of the SM relations is practically maintained, while the second is violated by less than 5%. However, the $\mathcal{U}_{db} = 0$ constraint may be violated by $\mathcal{O}(0.2)$ effects: it should be replaced by a unitarity *quadrangle*. A geometrical presentation of the new relation is given in Fig. 7. It should be stressed that, at present, only the *magnitudes* of U_{db} and U_{sb} are experimentally constrained, but not their phases. Each of the angles $\bar{\alpha}$ and $\bar{\beta}$ could be anywhere in the range $[0, 2\pi]$.

(*iii*) Z-mediated B decays.

Our main interest is in hadronic B^0 decays to CP eigenstates, where the quark sub-process is $\bar{b} \to \bar{u}_i u_i \bar{d}_j$, with $u_i = u, c$ and $d_j = d, s$. These processes get additional contributions from Z-mediated FCNC. The ratio between the magnitudes of the Z-mediated amplitude and the W-mediated amplitude is:

$$[(1/2) - (2/3)\sin^2\theta_W] \left|U_{jb}^*/(V_{ij} V_{ib}^*)\right| \approx (1/3) \left|U_{jb}^*/(V_{ij} V_{ib}^*)\right|. \tag{17.20}$$

To bound this ratio, we use the experimental constraints in eq. (17.12), our requirement that mixing of B_d mesons is dominated by Z-mediated FCNC in eq.

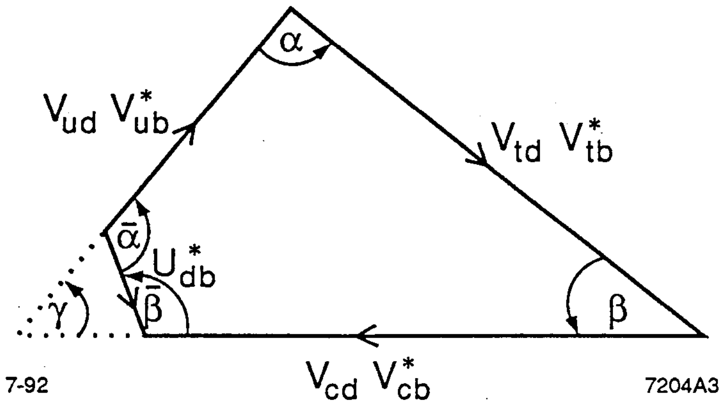

Figure 7. The unitarity quadrangle in a model with a fourth, $SU(2)_L$-singlet, charge $-1/3$ quark. V_{ij} are elements of the charged current mixing matrix while U_{db} is an element in the neutral current mixing matrix.

(17.16), and the range $0.07 \leq |V_{ub}/V_{cb}| \leq 0.13$. We find that the Z-mediated diagrams cannot dominate the relevant B decays. They can be safely neglected for $b \to s$ transitions, but may be significant for $b \to d$ (3–18%).

On the other hand, diagrams with no SM tree contributions [78] now have comparable contributions from penguin and Z-mediated tree diagrams.

(*iv*) New contributions to $\Gamma_{12}(B_q)$.

The difference in width comes from decay modes which are common to B_q and $\bar{B}_q$. As discussed above, there are new contributions to such decay modes from Z-mediated FCNC. It is important to note, however, that while the contributions to the difference in mass, M_{12}, are from tree level diagrams, namely $O(g^2)$, those to the difference in width, Γ_{12}, are still of $O(g^4)$. Consequently, no significant enhancement of the SM value for Γ_{12} is expected, and the relation $\Gamma_{12}(B_q) \ll M_{12}(B_q)$ is maintained.

In summary, the dominant mechanism for mixing in neutral B_d systems could be Z-mediated FCNC. The conditions for that are given in eq. (17.16). But the hadronic B decays of relevance are still dominated by SM W-mediated diagrams and $\Gamma_{12}(B_q) \ll M_{12}(B_q)$, so that CP asymmetries can be cleanly interpreted. Mixing in the B_s system cannot be dominated by Z-mediated FCNC. Mixing in the neutral K system can be dominated by Z-mediated FCNC if eq. (17.14) is satisfied. CP violation in the neutral K system can be dominated by Z-mediated FCNC if eq. (17.15) is satisfied.

17.4. AN EXPLICIT PARAMETRIZATION

It is convenient to use an explicit parametrization for the mixing matrices. We use the parametrization of refs. [79, 80] (appropriately modified to the 3×4 case). Assuming that all mixing angles θ_{ij} are small, we put $\cos\theta_{ij} = 1$. We use the following constraints from SM tree-level processes and from the unitarity of K:

$$\begin{aligned} s_{12} =& 0.22; \quad s_{23} \approx 0.04; \quad s_{13} \approx 0.004; \\ s_{14} \leq& 0.07; \quad s_{24} \leq 0.5. \end{aligned} \tag{17.21}$$

($s_{ij} \equiv \sin\theta_{ij}$.) We further assume that the unmeasured mixing angles fulfill the hierarchy $s_{14} < s_{24} < s_{34}$. More specifically, we assume that

$$q_{24} \equiv s_{24}/(s_{23}s_{34}), \quad q_{14} \equiv s_{14}/(s_{12}s_{23}s_{34}), \tag{17.22}$$

are both $\mathcal{O}(1)$. We remind the reader that a similar relation for the three generation mixing angles is experimentally verified:

$$q_{13} \equiv s_{13}/(s_{12}s_{23}) = 0.45 \pm 0.15. \tag{17.23}$$

Thus, V has the approximate form:

$$V = \begin{pmatrix} 1 & s_{12} & s_{13}e^{-i\delta_{13}} & s_{14}e^{-i\delta_{14}} \\ -s_{12} & 1 & s_{23} & s_{24}e^{-i\delta_{24}} \\ s_{12}s_{23} - s_{13}e^{i\delta_{13}} & -s_{23} & 1 & s_{34} \end{pmatrix}. \tag{17.24}$$

This gives for the relevant U_{pq} elements:

$$\begin{aligned}
U_{ds} =& s_{12}s_{23}^2 s_{34}^2 \left[(1 - q_{13}e^{-i\delta_{13}} - q_{24}e^{-i\delta_{24}} + q_{14}e^{i\delta_{14}})(1 - q_{24}e^{i\delta_{24}})\right], \\
U_{db} =& - s_{12}s_{23}s_{34}^2 \left[1 - q_{13}e^{-i\delta_{13}} - q_{24}e^{-i\delta_{24}} + q_{14}e^{-i\delta_{14}}\right], \\
U_{sb} =& s_{23}s_{34}^2 \left[1 - q_{24}e^{-i\delta_{24}}\right].
\end{aligned} \tag{17.25}$$

All the experimental constraints in eqs. (17.11) and (17.12) as well as the condition on $|U_{db}|$ in eq. (17.16) can be fulfilled with

$$s_{34}^2 \sim 0.04, \quad q_{24} \sim 1, \quad q_{14} \sim 3. \tag{17.26}$$

In this case, the dominant mechanism for B_d mixing will be the Z mediated FCNC, while B_s mixing is dominated by the Standard Model box diagram. On the other hand, we expect Im U_{ds} to be of the same order of magnitude as Re U_{ds}. Consequently, eq. (17.14) is not satisfied, so that ΔM_K gets no significant contributions from the Z-mediated FCNC, but eq. (17.15) may still be satisfied, in which case ϵ does get significant contributions from the Z mediated diagrams.

Eq. (17.25) implies that the phases in the mixing of B_d and B_s may depend on phases of the mixing matrix other than the single phase of the SM. This may give CP asymmetries which are very different from those predicted by the SM.

17.5. CP ASYMMETRIES IN B DECAYS

Our study involves the three types of CP asymmetries in B decays for which the direct decay is dominated by the W-mediated tree level diagram: $a_{\psi K_S}$, a_{DD} and $a_{\pi\pi}$. These asymmetries still arise almost purely from interference of mixing and decay. Furthermore, as the first unitarity relation is practically maintained, we still have (taking into account CP-parities)

$$a_{\psi K_S} = -a_{DD}. \tag{17.27}$$

However, as the dominant mechanism of mixing in the B system is the Z-

mediated tree level diagram,

$$\left(\frac{q}{p}\right)_B = \frac{U^*_{db}}{U_{db}}. \tag{17.28}$$

It is now straightforward to evaluate $\mathrm{Im}\lambda_{\psi K_S}$ and $\mathrm{Im}\lambda_{\pi\pi}$. We find that the various asymmetries simply measure angles of the unitarity quadrangle shown in Fig. 7:

$$a_{\psi K_S} = -a_{DD} = -\sin 2\bar{\beta}, \quad a_{\pi\pi} = -\sin 2\bar{\alpha}, \tag{17.29}$$

where

$$\bar{\alpha} \equiv \arg\left(\frac{V_{ud}V^*_{ub}}{U^*_{db}}\right); \quad \bar{\beta} \equiv \arg\left(\frac{U^*_{db}}{V_{cd}V^*_{cb}}\right). \tag{17.30}$$

The important point about the modification of the Standard Model predictions is not that the angles α, β and γ may have very different values from those predicted by the SM, but rather that the CP asymmetries do not measure these angles anymore.

As there are no experimental constraints on $\bar{\alpha}$ and $\bar{\beta}$ so that the full range $[0, 2\pi]$ is allowed for each of them, the full range $[-1, +1]$ is possible for each of the asymmetries. This is clearly seen when using the explicit parametrization given in eqs. (17.24) and (17.25):

$$\begin{aligned} \mathrm{Im}\ \lambda_{\psi K_S} &= -\ \mathrm{Im}\ \lambda_{DD} = -\mathrm{Im}\left[\frac{1 - q_{13}e^{i\delta_{13}} - q_{24}e^{i\delta_{24}} + q_{14}e^{i\delta_{14}}}{1 - q_{13}e^{-i\delta_{13}} - q_{24}e^{-i\delta_{24}} + q_{14}e^{-i\delta_{14}}}\right], \\ \mathrm{Im}\ \lambda_{\pi\pi} &= \mathrm{Im}\left[\frac{e^{-i\delta_{13}}(1 - q_{13}e^{i\delta_{13}} - q_{24}e^{i\delta_{24}} + q_{14}e^{i\delta_{14}})}{e^{i\delta_{13}}(1 - q_{13}e^{-i\delta_{13}} - q_{24}e^{-i\delta_{24}} + q_{14}e^{-i\delta_{14}})}\right]. \end{aligned} \tag{17.31}$$

It is rather obvious that our ignorance of the phases δ_{14} and δ_{24} allows any value for the various asymmetries. This model demonstrates that there exist extensions of the Standard Model where dramatic deviations from the Standard Model predictions for CP asymmetries in B decays are not unlikely.

Finally, let us mention an interesting point about this model. As mixing of the B_s system is dominated by the Standard Model process, we have, as in the Standard Model,

$$\left(\frac{q}{p}\right)_{B_s}\left(\frac{\bar{A}_{B_s\to\psi\phi}}{A_{B_s\to\psi\phi}}\right)\approx 0. \tag{17.32}$$

As shown in ref. [81], this is a sufficient condition for the angles extracted from $B\to\psi K_S$, $B\to\pi\pi$ and $B_s\to\rho K_S$ to sum up to π. This happens in spite of the fact that the first two measurements do not correspond to β and α of the unitarity triangle.

18. Extending the Scalar Sector

Neutral Higgs Exchange

18.1. INTRODUCTION

CP violation could appear in neutral scalar exchange in models with at least two Higgs doublets [82, 83]. If we require both spontaneous *CP* violation and NFC then at least three scalar doublets [84] (or two doublets and a singlet) are required. (For a discussion of *CP* violation in multi-scalar models and no NFC, see refs. [85 – 88].)

We denote scalar doublets by Φ_i, with

$$\Phi_i=\sqrt{\tfrac{1}{2}}\begin{pmatrix}\phi_i^+\\ \phi_i^0\end{pmatrix},\quad \phi_i^0=v_i+R_i+iI_i. \tag{18.1}$$

The normalization of the VEVs v_i is such that

$$v^2\equiv\sum_{i=1}^{k}v_i^2=(\sqrt{2}G_F)^{-1}\approx(246\ GeV)^2. \tag{18.2}$$

We assume NFC with only Φ_1 coupled to D_R and only Φ_2 coupled to U_R:

$$-\mathcal{L}_Y = \overline{Q^I_{Li}} G_{ij} \Phi_1 d^I_{Rj} + \overline{Q^I_{Li}} F_{ij} \Phi_2 u^I_{Rj} + \text{h.c.}. \tag{18.3}$$

The quark mass matrices are then

$$M_d = \sqrt{\tfrac{1}{2}} G\ v_1, \quad M_u = \sqrt{\tfrac{1}{2}} F\ v_2. \tag{18.4}$$

The neutral Higgs interaction with quark mass eigenstates is

$$-\mathcal{L}^n_Y = \frac{R_1}{v_1} \bar{D} M_d^{\text{diag}} D + \frac{I_1}{v_1} \bar{D} M_d^{\text{diag}} i\gamma_5 D + \frac{R_2}{v_2} \bar{U} M_u^{\text{diag}} U + \frac{I_2}{v_2} \bar{U} M_u^{\text{diag}} i\gamma_5 U. \tag{18.5}$$

We now rotate to the scalar mass eigenbasis,

$$\begin{pmatrix} H_1^0 \\ H_2^0 \\ \cdot \\ \cdot \\ \cdot \\ G^0 \end{pmatrix} = O \begin{pmatrix} R_1 \\ I_1 \\ R_2 \\ I_2 \\ \cdot \\ \cdot \end{pmatrix}, \tag{18.6}$$

where G^0 is the would-be Goldstone boson eaten by the Z^0. The Yukawa Lagrangian for neutral Higgs, bottom and top mass eigenstates is given by [89]

$$-\mathcal{L}^n_Y = \sum_{i=1}^{2k-1} \left[(m_b/v_1) \bar{b} (O_{i1} + i\gamma_5 O_{i2}) b + (m_t/v_2) \bar{t} (O_{i3} + i\gamma_5 O_{i4}) t \right] H_i^0, \tag{18.7}$$

and similarly to other quarks. (Another common notation in the literature is

$$g_{1i} = -\frac{v}{v_1} O_{i1}, \quad g_{2i} = -\frac{v}{v_1} O_{i2}, \quad g_{3i} = -\frac{v}{v_2} O_{i3}, \quad g_{4i} = -\frac{v}{v_2} O_{i4}.) \tag{18.8}$$

CP violation in the neutral Higgs sector comes from mixing of CP even and

CP odd fields. The quantities that appear in CP violating observables are

$$\begin{aligned} \text{Im } A_1(q) &= \frac{2}{v_1^2} \sum_i \frac{O_{i1} O_{i2}}{q^2 - m_{H_i}^2}, \\ \text{Im } A_2(q) &= \frac{2}{v_2^2} \sum_i \frac{O_{i3} O_{i4}}{q^2 - m_{H_i}^2}. \end{aligned} \tag{18.9}$$

There are two more CP violating quantities, $g_{1i}g_{4i}$ and $g_{2i}g_{3i}$, which correspond to combinations of A_0 and $\tilde{A}_0$ in ref. [90]. Dimensionless quantities Z_j are defined through

$$A_j(q^2) = \sum_i \frac{\sqrt{2} G_F Z_j}{q^2 - m_{H_i}^2}. \tag{18.10}$$

It has been shown [90, 91] that in a two doublet model, there is a unitarity bound,

$$\begin{aligned} |\text{Im} Z_1| &= 2 |\sum_{i=1}^{3} g_{1i} g_{2i}| \le \left| \frac{v_u}{v_d} \right| \left(1 + \left| \frac{v_u}{v_d} \right|^2 \right)^{1/2} = \begin{cases} \sqrt{2} & |v_d| = |v_u|, \\ |v_u/v_d|^2 & |v_d| \ll |v_u|, \end{cases} \\ |\text{Im} Z_2| &= 2 |\sum_{i=1}^{3} g_{3i} g_{4i}| \le \left| \frac{v_d}{v_u} \right| \left(1 + \left| \frac{v_d}{v_u} \right|^2 \right)^{1/2} = \begin{cases} \sqrt{2} & |v_d| = |v_u|, \\ |v_d/v_u| & |v_d| \ll |v_u|. \end{cases} \end{aligned} \tag{18.11}$$

It was further shown that a plausible value of the couplings is close to this unitarity bound [90, 92].

18.2. CP VIOLATION IN NEUTRAL MESON DECAYS

With NFC, neutral Higgs exchange cannot mediate flavor changing processes. Thus it cannot contribute to ϵ at $\mathcal{O}(G_F^2)$ and to ϵ'/ϵ at $\mathcal{O}(G_F)$. Similarly, it cannot contribute significantly to either $B - \bar{B}$ mixing or B decays. Neutral Higgs exchange in models that incorporate NFC is then irrelevant for CP violation in neutral meson systems.

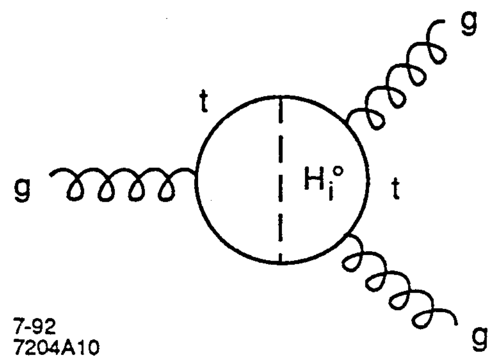

Figure 8. A contribution to the three gluon operator in a two scalar doublet model.

A two loop diagram involving a top quark and neutral Higgs (see Fig. 8) would contribute to $\mathcal{D}_N$ through the three gluon operator [45, 93],

$$C = \frac{2\sqrt{2}G_F g_s^3}{(4\pi)^4}\mathrm{Im}Z_2 h(m_t, m_H), \tag{18.12}$$

so that

$$\mathcal{D}_N \sim 4 \times 10^{-21} e\zeta \mathrm{Im}Z_2 h(m_t, m_H) \text{ cm.} \tag{18.13}$$

ζ is a QCD correction factor [94],

$$\zeta = \left[\frac{g_s(\mu)}{g_s(m_t)}\right]^{-108/23}\left[\frac{g_s(\mu)}{4\pi}\right]^3 \sim 10^{-4}. \tag{18.14}$$

The function $h(m_t, m_H)$ is a result of the two loop integration [93],

$$h(m_t, m_H) = \frac{m_t^4}{4}\int_0^1 dx \int_0^1 du \frac{u^3 x^3 (1-x)}{[m_t^2 x(1-ux) + m_H^2(1-u)(1-x)]^2}. \tag{18.15}$$

For m_H not much larger than m_t, $h(m_t, m_H) = \mathcal{O}(0.1)$. If, furthermore, Im$Z_2$ is indeed close to the unitarity bound (18.11), then

$$\mathcal{D}_N \sim 4 \times 10^{-26} |v_d/v_u| \; e \text{ cm}, \tag{18.16}$$

for $|v_d| \ll |v_u|$, and even larger ($\sim 6 \times 10^{-26}$ e cm) for $|v_d| = |v_u|$. Other operators induced by neutral Higgs exchange give contributions comparable to (18.16) [95, 42].

Neutral Higgs exchange could also contribute to $\mathcal{D}_e$, the EDM of the electron. Two loop diagrams may induce values close to the experimental bound [96 – 99].

18.4. SUMMARY

CP violation from neutral Higgs exchange in models with NFC is negligible for the neutral kaon system. It could however give $\mathcal{D}_N$ (and $\mathcal{D}_e$) close to the experimental upper bound.

It was recently realized that, due to the large Yukawa coupling of the top quark, neutral Higgs exchange could induce interesting *CP* violating phenomena in top physics [100].

Charged Scalar Exchange

18.5. INTRODUCTION

CP violation could arise in charged scalar exchange if there are at least three Higgs doublets [83]. This is also the minimal number of doublets required when *CP* breaking is spontaneous only and NFC is maintained [84]. In this case, $\delta_{KM} = 0$ and all *CP* violation comes from the mixing of scalar fields. It is, of course, possible that *CP* is explicitly broken, in which case both quark and Higgs mixings provide *CP* violation.

The charged Higgs interaction with quark mass eigenstates is

$$-\mathcal{L}_Y^c = -\frac{\phi_1^+}{v_1}\bar{U}_L V M_d^{\rm diag} D_R + \frac{\phi_2^+}{v_2}\bar{U}_R M_u^{\rm diag} V D_L + {\rm h.c.}. \qquad (18.17)$$

We now rotate to the scalar mass eigenbasis,

$$\begin{pmatrix} H_1^+ \\ H_2^+ \\ \cdot \\ \cdot \\ G^+ \end{pmatrix} = Y \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}, \qquad (18.18)$$

where G^+ is the would-be Goldstone boson eaten by the W^+. The Lagrangian for charged Higgs mass eigenstates coupling to quark mass eigenstates is

$$\mathcal{L}_Y^c = \frac{1}{v}\sum_{j=1}^{k-1}(\alpha_j \overline{U_L} V M_d^{\rm diag} D_R + \beta_j \overline{U_R} M_u^{\rm diag} V D_L) H_j^+ + {\rm h.c.}, \qquad (18.19)$$

where

$$\alpha_j = -\frac{v}{v_1} Y_{j1}^* = -\frac{Y_{j1}^*}{Y_{k1}^*}, \quad \beta_j = -\frac{v}{v_2} Y_{j2}^* = -\frac{Y_{j2}^*}{Y_{k2}^*}. \qquad (18.20)$$

CP violation in the charged Higgs sector comes from phases in the mixing matrix for charged scalars (and requires, therefore, at least three doublets). The quantity that appears in *CP* violating observables is

$$\text{Im } A(q) = 2\sqrt{2} G_F \sum_{i=1}^{2} \frac{\text{Im}\alpha_i \beta_i^*}{q^2 - m_{H_i^+}^2}. \qquad (18.21)$$

With only two physical charged scalars, there is only one *CP* violating parameter in the charged Higgs sector,

$$\mathrm{Im}Z = 2\mathrm{Im}(\alpha_1\beta_1^*) = -2\mathrm{Im}(\alpha_2\beta_2^*), \tag{18.22}$$

where, again, a dimensionless quantity Z was defined through

$$A(q^2) = \sum_i \frac{\sqrt{2}G_F Z_i}{q^2 - m^2_{H_i^+}}. \tag{18.23}$$

There is an interesting question of whether charged Higgs exchange could be the *only* source of CP violation. In other words, we would like to know whether a model of extended Higgs sector with spontaneous CP violation and natural flavor conservation is viable. The answer has been subject to controversy [101 − 104]. In recent years, there have been two attempts [105, 106] to show that this possibility is not yet ruled out. We repeat the analysis (incorporating new data) and find that CP violation cannot result from charged Higgs exchange only; thus confirming the conclusion of ref. [107].

18.6. THE ϵ PARAMETER

In this framework, neither short distance contributions nor long distance contribution from an intermediate 2π-state can produce large enough ϵ. Thus, to account for ϵ, one needs to assume that the dominant contribution comes from an intermediate η_0 (the $SU(3)$-singlet component of the pseudoscalar nonet):

$$\epsilon \approx \frac{e^{i\pi/4}}{\sqrt{2}\Delta M_K}\mathrm{Im}\frac{\langle K^0 | H_{\Delta S=1} |\eta_0\rangle \langle \eta_0 | H_{\Delta S=1} |\bar{K}^0\rangle}{m_K - m_{\eta_0}}. \tag{18.24}$$

We followed the analyses of refs. [105, 106].* We find that, to account for the

* We were unable to reproduce the result of eq. (16) of ref. [106]. It seems to us that they may have used a numerically wrong value for $\langle K^0 | H | \pi^0\rangle$. In ref. [105], in their definition of $G(x)$, there is an overall factor of $1/(1-x)$ missing. This may have enhanced the top contribution in their calculation.

experimental value of ϵ, the Higgs parameters should fulfill

$$\frac{\mathrm{Im}Z}{2m_H^2}\left[\ln\frac{m_H^2}{m_c^2}-\frac{3}{2}\right]=0.11\ GeV^{-2}. \tag{18.25}$$

With $m_{H^+}\geq 42\ GeV$, this gives

$$\mathrm{Im}Z \gtrsim 80. \tag{18.26}$$

18.7. $\mathcal{D}_N$

A large contribution to $\mathcal{D}_N$ comes from the electric dipole moment of the down quark:

$$\mathcal{D}_N=\frac{\sqrt{2}g_G e}{9\pi^2}m_d\mathrm{Im}(\alpha\beta^*)\left[\bar{\eta}_c|V_{cd}|^2\bar{g}\left(\frac{m_c^2}{m_H^2}\right)+\bar{\eta}_t|V_{td}|^2\bar{g}\left(\frac{m_t^2}{m_H^2}\right)\right], \tag{18.27}$$

with

$$g(x)=\frac{x}{(1-x)^2}\left[\frac{5x}{4}-\frac{1-3x/2}{1-x}\ln x-\frac{3}{4}\right]. \tag{18.28}$$

We neglect the contribution of the top quark (it adds to the charm contribution) and we take the current mass at 1 GeV for m_d ($m_d=9\ MeV$). It is more plausible that we should actually use the constituent $m_d\approx 330\ MeV$.† Thus, we may be underestimating $\mathcal{D}_N$ by as much as a factor ~ 40. The result is

$$\mathcal{D}_N\approx 2.5\times 10^{-25}\ e\ \mathrm{cm}. \tag{18.29}$$

We conclude that in a model where ϵ arises from charged Higgs exchange, $\mathcal{D}_N$ is at least two times larger and more probably two orders of magnitude larger than the experimental upper bound.

† See a discussion of this point in ref. [108].

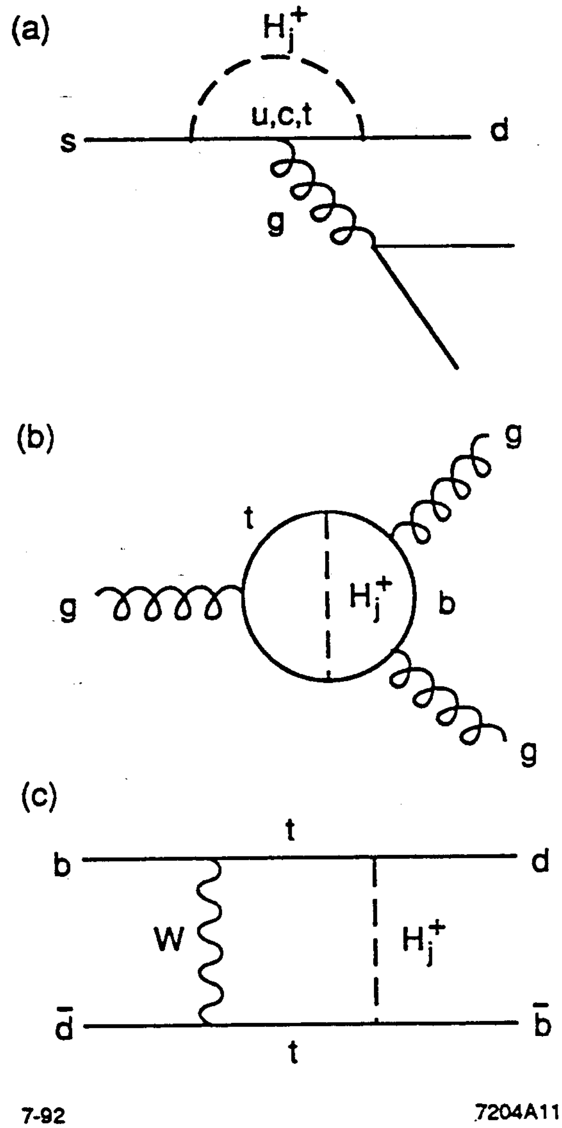

Figure 9. CP violating contributions from charged scalar exchange: (a) A contribution to $\Delta S = 1$ processes. It affects both ϵ and ϵ'. (b) A contribution to the three gluon operator. (c) A contribution to $B - \bar{B}$ mixing.

CP violation in the charged Higgs sector would also contribute to the three gluon operator with [93]

$$C = 4\sqrt{2}\zeta G_F g_s^3 (4\pi)^{-4} \mathrm{Im}(\alpha\beta^*) h'(m_b, m_t, m_H), \tag{18.30}$$

where h' is a function of m_b, m_t and m_H which, for $m_t \gg m_b$ is given by

$$h'(m_t \gg m_b) = \frac{m_t^2 m_H^4}{4(m_H^2 - m_t^2)^3} \left[\ln \frac{m_H^2}{m_t^2} - \frac{3}{2} + 2\frac{m_t^2}{m_H^2} - \frac{1}{2}\frac{m_t^2}{m_H^4} \right]. \tag{18.31}$$

For $m_H^2 \ll m_t^2$, $h' \approx h/2$, while for $m_H^2 \gg m_t^2$, $h' \approx h$. The QCD correction factor ζ is given by [109, 110]

$$\zeta = \left[\frac{g_s(m_b)}{g_s(m_t)}\right]^{\gamma_b/\beta_5} \left[\frac{g_s(m_c)}{g_s(m_b)}\right]^{\gamma_g/\beta_4} \left[\frac{g_s(\mu)}{g_s(m_c)}\right]^{\gamma_g/\beta_3}, \tag{18.32}$$

where $\gamma_g = -18$, $\gamma_b = -14/4$ and $\beta_n = (33 - 2n)/6$. It follows then from (18.25) that, if charged Higgs exchange accounts for ϵ,

$$\mathcal{D}_N \approx 10^{-23} \; e \text{ cm}, \tag{18.33}$$

two orders of magnitude above the experimental upper bound.

18.8. ϵ'/ϵ

Early calculations of ϵ'/ϵ, using PCAC relations for the physical $K_L \to 2\pi$ amplitude, found $\epsilon'/\epsilon \approx -1/22$ [101, 102]. It was later realized [103] that actually the contribution to $\mathrm{Im}A(K^0 \to \pi\pi)$ is chirally suppressed,

$$\langle \pi^+\pi^- | \mathcal{L}_- | K^0 \rangle = -\frac{i\sqrt{2}D}{2f_\pi} \langle \pi^0 | \mathcal{L}_- | K^0 \rangle . \tag{18.34}$$

The suppression factor D is expected to be of $\mathcal{O}(m_K^2/4\pi^2 f_\pi^2)$, and leads to ϵ'/ϵ of $\mathcal{O}(10^{-3})$. Note that in this framework the value of ϵ'/ϵ is independent of $\langle \pi | \mathcal{L}_- | K \rangle$ and consequently of the CP violating parameters of the Higgs sector.

18.9. CP ASYMMETRIES IN B DECAYS

The Y-matrix introduces new phases into the charged scalar couplings to quarks. However, the leading contribution from ϕ_j^+-exchange diagrams to $B-\bar{B}$ mixing comes from the term proportional to m_t. This gives $(Y_{j2}^* V_{td})(Y_{j2}^* V_{tb})^*$, and has exactly the same phase as the Standard Model W-exchange box diagram. Consequently, $(q/p)_B = (M_{12}^*/M_{12})$ remains unchanged, and there is no modification to the Standard Model predictions for CP asymmetries in B decays [111]. Note that this conclusion is independent of whether charged scalar exchange contributes significantly to $B-\bar{B}$ mixing or not.

18.10. SUMMARY

A relative phase between VEVs in a multi-Higgs model with NFC *cannot* be the only source of CP violation. Of course, in a model with explicit CP violation, such that $\delta_{KM} \neq 0$, a relative phase between VEVs could be an additional source of CP violation. It would not affect ϵ significantly, but it may saturate the upper bound on $\mathcal{D}_N$. An order of magnitude estimate suggests that if the contribution to ϵ is small, so is the contribution to ϵ'/ϵ independently of the Higgs parameters. There is no effect on CP asymmetries in B decays.

19. Extending the Gauge Sector

Left-Right Symmetry (LRS)

19.1. INTRODUCTION

We study a specific version of LRS models, where P, C and CP are symmetries of the Lagrangian that are spontaneously broken [112 – 117]. The electroweak gauge group is $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Left-handed quarks reside in $(2,1)_{1/3}$ representations and right-handed ones in $(1,2)_{1/3}$. The scalar content [118] of the minimal LRS model is $\Phi(2,2)_0$, $\Delta_L(3,1)_2$ and $\Delta_R(1,3)_2$. A model with only minimal scalar content and spontaneous CP violation predicts unacceptably large FCNC [119]. To avoid this, one has to add scalar singlets or triplets but these do not affect our analysis. The only specific assumption about the scalar sector that we make is the existence of a single Φ-field. (At least one such field is necessary to induce fermion masses.) The VEV of Φ is

$$\langle \Phi \rangle = \begin{pmatrix} k & 0 \\ 0 & k' e^{i\eta} \end{pmatrix}. \tag{19.1}$$

The relative phase η between k and k' spontaneously breaks CP : in principle, it is the only source of CP violation in this model. Eventually there are seven CP violating phases in the mass eigenbasis. They all vanish when $\eta = 0$ but they are independent parameters.

The phase η appears explicitly in the mixing of the charged gauge bosons:

$$\begin{aligned} W_1 &= \cos\xi\ W_L + e^{-i\eta} \sin\xi\ W_R, \\ W_2 &= -\ e^{i\eta} \sin\xi\ W_L + \cos\xi\ W_R, \end{aligned} \tag{19.2}$$

where

$$\xi = \frac{kk'}{v_R^2}. \tag{19.3}$$

The Yukawa couplings are given by

$$\mathcal{L}_{\text{Yukawa}} = \overline{Q_L^I}(A\Phi + B\tau_2\Phi^*\tau_2)Q_R^I + \text{h.c.}, \tag{19.4}$$

where $Q^I_{L(R)}$ are quark doublets of $SU(2)_{L(R)}$, τ_2 is the Pauli matrix acting in the $SU(2)_L$ or $SU(2)_R$ space, A and B are matrices in generation space.

P symmetry requires that A and B are hermitian; C symmetry requires that A and B are symmetric; and CP invariance implies that A and B are real. The mass matrices,

$$\begin{aligned} M_u =& kA + k' e^{-i\eta} B, \\ M_d =& k' e^{i\eta} A + kB, \end{aligned} \tag{19.5}$$

are symmetric. The symmetry of the mass matrices implies

$$V_R = F_u V_L^* F_d^\dagger, \tag{19.6}$$

where V_L and V_R are the mixing matrices for left-handed and right-handed quarks, respectively, and F_u and F_d are diagonal unitary matrices:

$$F_u = \mathrm{diag}(e^{i\phi_u}, e^{i\phi_c}, e^{i\phi_t}); \quad F_d = \mathrm{diag}(e^{i\phi_d}, e^{i\phi_s}, e^{i\phi_b}). \tag{19.7}$$

On top of the single CP violating phase of V_L there are 5 CP violating phase differences in F_u and F_d.

For the purpose of studying *new* contributions to ϵ, $\mathcal{D}_N$ and ϵ'/ϵ, it is simpler to work in a two generation framework. In this case V_L is real and there are three phases in F_u and F_d. We define:

$$\begin{aligned} \gamma =& (\phi_c + \phi_u - \phi_s - \phi_d)/2 + \eta, \\ \delta_1 =& (\phi_c - \phi_u + \phi_s - \phi_d)/2, \\ \delta_2 =& (\phi_c - \phi_u - \phi_s + \phi_d)/2. \end{aligned} \tag{19.8}$$

Choosing a basis where V_L is real and the mixing of $W_L - W_R$ is real, these phases appear in V_R only:

$$V_W = \begin{pmatrix} c_\xi & s_\xi \\ -s_\xi & c_\xi \end{pmatrix}; \; V_L = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix}; \; V_R = e^{i\gamma} \begin{pmatrix} e^{-i\delta_2} c_\theta & e^{-i\delta_1} s_\theta \\ -e^{i\delta_1} s_\theta & e^{i\delta_2} c_\theta \end{pmatrix}. \tag{19.9}$$

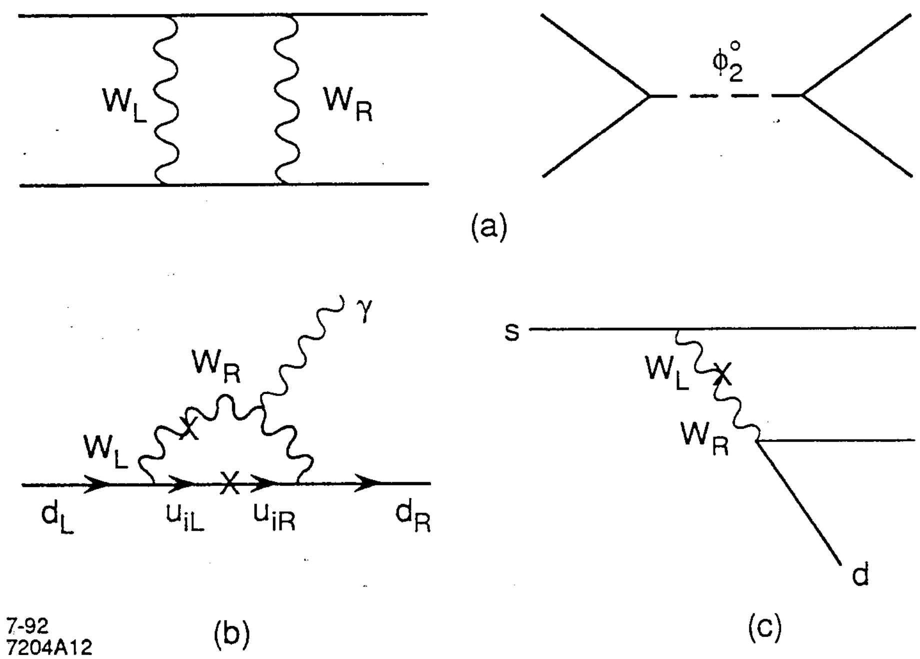

Figure 10. *CP* **violating diagrams in a LRS framework. (a) Contributions to ϵ from a box diagram with W_R and from a tree diagram with neutral scalar. (b) A contribution to the EDM of the down quark. (c) A contribution to ϵ'.**

19.2. THE ϵ PARAMETER

For ϵ, the dominant contributions come from the diagrams of Fig. 10(a). $W_L - W_R$ mixing can be safely neglected. The value of $M_{12}(K)$ in this model is [114, 117]

$$\frac{M_{12}^{\rm LRS}}{M_{12}^{\rm SM}} = 1 - e^{i(\delta_2-\delta_1)}\left[430\beta - 15\beta\ln\beta + Q_H^2\left(11600\beta_H - 15\beta_H\ln\beta_H\right)\right], \quad (19.10)$$

where

$$\beta = \frac{m_{W_1}^2}{m_{W_2}^2}, \quad \beta_H = \frac{m_{W_1}^2}{m_{H^0}^2}, \quad Q_H = \frac{k^2 + k'^2}{k^2 - k'^2}, \quad (19.11)$$

and we assumed $m_{H^0} \sim m_{A^0} \sim m_{H^+}$. The factor of 430 was first calculated in ref. [120], and is the product of three factors of $\mathcal{O}(1)$: a factor of 2 since two diagrams contribute, a factor of $4[\ln(m_{W_1}^2/m_c^2)-1] \sim 28$ from loop integration and a factor of 7.6 due to the Lorentz structure of the relevant matrix element. The factor of 11600 arises because H^0 contributes at tree level. Requiring $2\mathrm{Re}M_{12}^{\mathrm{LRS}} \leq \Delta M_K$ gives

$$m_{W_2} \geq 1.7\ TeV, \quad m_H \geq 8.8\ TeV. \tag{19.12}$$

Note that the bound $\beta \lesssim 1/430$ implies

$$\xi \leq 2.2 \times 10^{-3}. \tag{19.13}$$

Eq. (19.10) leads to

$$|\epsilon| = \frac{\sin(\delta_2 - \delta_1)}{2\sqrt{2}} \left[430\beta - 15\beta \ln\beta + Q_H^2 \left(11600\beta_H - 15\beta_H \ln\beta_H\right)\right]. \tag{19.14}$$

To derive an upper bound on $\sin(\delta_2 - \delta_1)$, we take $m_H^2/m_{W_2}^2 \leq 4\pi/g_R^2 \sim 30$. Then

$$\beta \sin(\delta_2 - \delta_1) \lesssim \frac{2\sqrt{2}|\epsilon|}{820} \sim 10^{-5} \Longrightarrow \sin(\delta_2 - \delta_1) \leq \left(\frac{m_{W_2}}{30\ TeV}\right)^2. \tag{19.15}$$

Conversely, for ϵ to be dominated by the LRS contribution, we need (assuming $m_H \gtrsim m_{W_2}$)

$$\beta \sin(\delta_2 - \delta_1) \gtrsim \frac{2\sqrt{2}|\epsilon|}{12000} \sim 5 \times 10^{-7} \Longrightarrow m_{W_2} \lesssim 120\ TeV\ [\sin(\delta_2 - \delta_1)]^{1/2}. \tag{19.16}$$

19.3. $\mathcal{D}_N$

The most important LRS contributions to $\mathcal{D}_n$ arise from quark EDMs (see Fig. 10(b)). All phases in V_R contribute to $\mathcal{D}_N$ [121], but $(\gamma + \delta_1)$ which contributes to $\mathcal{D}_d$ proportionally to m_c is the most important one. A recent calculation [122] gives

$$\mathcal{D}_N = 2\times 10^{-23}\xi[4.5\sin(\gamma-\delta_2)+74\sin(\gamma+\delta_1)-1.1\sin(\gamma-\delta_1)+16\sin(\gamma+\delta_2)]e \text{ cm}. \tag{19.17}$$

The upper bound (19.13) implies $\mathcal{D}_N \leq 4\times 10^{-24}$ e cm. Assuming no strong cancellations among the various terms in (19.17), we get

$$\xi\sin(\gamma+\delta_1) \lesssim 10^{-4}. \tag{19.18}$$

There are also LRS contributions to $\mathcal{D}_n$ through the three gluon operator. These contributions are estimated to be an order of magnitude smaller than those from the quark EDMs [123].

19.4. ϵ'/ϵ

The contribution to ϵ'/ϵ (Fig. 10(c)) comes from all phases in V_R but the phases in the first row, $(\gamma-\delta_1)$ and $(\gamma-\delta_2)$, contribute at tree level with $W_L - W_R$ mixing [112, 113]. A recent calculation [122] gives

$$|\epsilon'/\epsilon| = 276\xi|\sin(\gamma-\delta_2)+\sin(\gamma-\delta_1)-0.1\sin(\gamma+\delta_1)-0.1\sin(\gamma+\delta_2)|. \tag{19.19}$$

The bound (19.13) implies $|\epsilon'/\epsilon| \leq 1.3$. Assuming no strong cancellations among the various terms in (19.19), we get

$$\xi[\sin(\gamma-\delta_1)+\sin(\gamma-\delta_2)] \lesssim 10^{-5}. \tag{19.20}$$

19.5. CP ASYMMETRIES IN B DECAYS

The effect of LRS on CP asymmetries in B decays is very small because LRS contributions to $B-\bar{B}$ mixing are small in magnitude. The reason for that is as follows. One of the enhancement factors for LRS contribution to $K-\bar{K}$ mixing is the hadronic matrix element,

$$\frac{\langle K^0|\,\bar{d}_L s_R \bar{d}_R s_L\,|\bar{K}^0\rangle}{\langle K^0|\,(\bar{d}_L\gamma^\mu s_L)^2\,|\bar{K}^0\rangle} = \frac{3}{4}\left[\left(\frac{m_K}{m_s+m_d}\right)^2+\frac{1}{6}\right] \approx 7.6. \tag{19.21}$$

However, as $m_B \approx m_b$, there is no similar enhancement in the B system. (The corresponding factor in B is very close to 1.) This implies that if LRS contributions to $K-\bar{K}$ mixing are as large as the Standard Model contribution, then the LRS contributions to $B-\bar{B}$ mixing are $\mathcal{O}(0.1)$ of the Standard Model contribution.

19.6. SUMMARY

Even though all the phases in the LRS model with spontaneously broken CP arise from the single phase η in the VEV $\langle\Phi\rangle$, it is difficult to relate their values unless one makes additional assumptions. Thus, the three bounds that we found could all be saturated simultaneously [124]:

$$\begin{aligned} |\beta\sin(\delta_2-\delta_1)| &\lesssim 10^{-5}, \\ |\xi\sin(\gamma+\delta_1)| &\lesssim 10^{-4}, \\ |\xi[\sin(\gamma-\delta_1)+\sin(\gamma-\delta_2)]| &\lesssim 10^{-5}. \end{aligned} \tag{19.22}$$

However, without (at least mild) fine-tuning, saturation of the ϵ'/ϵ bound would imply that the contribution to $\mathcal{D}_N$ is one to two orders of magnitude below the present experimental limit. If $k'/k \leq 0.1$ and all phases are of the same order of magnitude, then the ϵ-bound is the strongest.

For $k'/k \ll 1$, one can find relations among the various phases:

$$\delta_2 = -\frac{1}{2}\frac{k'}{k}\frac{m_c}{m_s}\sin\eta, \quad \delta_1 = -3\delta_2, \quad \gamma = \eta - \delta_2. \tag{19.23}$$

If, furthermore, $k'/k \ll m_s/m_c$, then (19.22) gives

$$|\xi(m_c/m_s)\sin\eta| \lesssim 10^{-5}, \quad |\xi\sin\eta| \lesssim 10^{-4}, \quad |\xi\sin\eta| \lesssim 5\times10^{-6}, \tag{19.24}$$

namely, the ϵ-bound is the strongest. Furthermore, ϵ'/ϵ and $\mathcal{D}_N$ are related in this case [113, 122]

$$|\mathcal{D}_N| = 3.6\times10^{-24}|\epsilon'/\epsilon|\ e\ \text{cm}. \tag{19.25}$$

20. SUSY

20.1. SOURCES OF CP VIOLATION IN MINIMAL SUSY

CP violation in SUSY theories has been the subject of intensive theoretical study [125 – 133]. Our discussion here follows for the most part the very clear discussion in ref. [126].

The simplest and most predictive among SUSY models is the low energy effective theory of the minimal $N=1$ supergravity. The low energy gauge group is $SU(3)\times SU(2)\times U(1)$. There are three generations of left chiral matter fields, $Q(3,2)_{1/6}$, $\bar{U}(\bar{3},1)_{-2/3}$, $\bar{D}(\bar{3},1)_{1/3}$, $L(1,2)_{-1/2}$, $\bar{E}(1,1)_1$, and a pair of Higgs supermultiplets, $H_u(1,2)_{1/2}$ and $H_d(1,2)_{-1/2}$. The Yukawa couplings and scalar potential in the supersymmetric limit are derived from the superpotential,

$$W = \bar{U}\lambda_U Q H_u + \bar{D}\lambda_D Q H_D + \bar{E}\lambda_E L H_D + \mu H_u H_d. \tag{20.1}$$

The λ_i are general 3×3 matrices. The SUSY breaking is due to the hidden sector and gives rise to three types of soft SUSY breaking operators:

(i) Trilinear scalar self couplings (ξ_i are general 3×3 matrices):

$$[\bar{U}\xi_U Q H_u + \bar{D}\xi_D Q H_d + \bar{E}\xi_E L H_d + \mu B H_u H_d] + \text{h.c.}. \tag{20.2}$$

(ii) Gaugino Majorana masses:

$$\tfrac{1}{2}M_1\lambda_1\lambda_1 + \tfrac{1}{2}M_2\lambda_2\lambda_2 + \tfrac{1}{2}M_3\lambda_3\lambda_3 + \text{h.c.} \tag{20.3}$$

(iii) Masses for the scalar fields z_a of the chiral superfields

$$M^2_{ab} z^*_a z_b + \text{h.c.} \tag{20.4}$$

It became a common practice to restrict

$$M^2_{ab} = m^2_{3/2}\delta_{ab} \tag{20.5}$$

at the renormalization point of the Planck scale. This is essentially a phenomenological requirement: in order that the contribution from box diagrams with squarks and winos does not exceed the measured value of ΔM_K, one needs

$$\left(V^\dagger \frac{M^2_Q}{m^2_{3/2}} V\right)_{12} \lesssim \frac{1}{100}\frac{m_{3/2}}{M_W}. \tag{20.6}$$

(M^2_Q is the mass matrix (20.5) for the scalar partners of left-handed quarks; V is the CKM matrix.) One could also implement (20.5) as a requirement on the properties of the Kähler potential. A second phenomenological constraints is that, if we write

$$\xi_i = m_{3/2} A\lambda_i + \tilde{\xi}_i, \tag{20.7}$$

then $\tilde{\xi}_i$ are small. Otherwise, large contributions to ΔM_K from strong superbox diagrams with LR current structure arise. If the superpotential is separable into a hidden sector piece (which breaks SUSY) and observable sector piece, then $\tilde{\xi} = 0$. We put

$$\tilde{\xi} = 0, \tag{20.8}$$

and assume grand unification,

$$M_1 = M_2 = M_3. \tag{20.9}$$

Then the theory at the Planck scale can be written as

$$\begin{aligned}
&[\bar{U}\lambda_U Q H_u + \bar{D}\lambda_D Q H_D + \bar{E}\lambda_E L H_D + \mu H_u H_d]_F + \text{h.c.} \\
&+m_{3/2}[A\bar{U}\lambda_U Q H_u + A\bar{D}\lambda_D Q H_D + A\bar{E}\lambda_E L H_D + \mu B H_u H_d]_A + \text{h.c.} \\
&+\tfrac{1}{2}\tilde{m}(\lambda_1\lambda_1 + \lambda_2\lambda_2 + \lambda_3\lambda_3) + \text{h.c.} + m_{3/2}^2 z_a^* z_a.
\end{aligned} \tag{20.10}$$

Note that even with the constraints (20.5), (20.8) and (20.9) imposed at the Planck scale, they do not hold at low energy. The RGEs generate flavor changing and CP violating contributions to the squark mass matrices and to the trilinear scalar self couplings. The crucial point in analyses of FCNC and CP violation in minimal SUSY is that the deviation of the mass matrices for down squarks from a unit matrix is almost negligible for $\tilde{D}_R$ while it is $\propto M_u^\dagger M_u$ for $\tilde{D}_L$:

$$\Delta M_Q^2 \approx \frac{m_{3/2}^2 \ln(M_P^2/m_W^2)}{8\pi^2}(3 + |A|^2)\lambda_U^\dagger \lambda_U. \tag{20.11}$$

This relates FCNC and CP violation in the K^0 and B^0 systems to the CKM parameters. Let us count the number of CP violating phases in (20.10).

- While λ_E can be made real and diagonal, there is one unremovable phase in λ_U and λ_D which we call δ_P. This is the usual KM phase, with a subscript P to denote the renormalization point of the Planck scale.

• The strong CP parameter now gets contributions from gaugino masses:

$$\bar{\theta} = \theta - \arg\det\lambda_U\lambda_D - 3\arg\tilde{m}. \tag{20.12}$$

• There are four more complex parameters: A, B, μ and $\tilde{m}$. But two of these phases are removable. Thus, in the low energy supersymmetric model described by (20.10), there are two new phases beyond the Standard Model:

$$\phi_A = \arg(A\tilde{m}^*), \quad \phi_B = \arg(B\tilde{m}^*), \tag{20.13}$$

(where we fixed the phase of μ to give $\arg(\mu B)=0$). The simplest hidden sector yields $\phi_A = \phi_B = 0$.

To summarize, SUSY effects on CP violation are of three types:

a. The values of CKM parameters deduced from experiments may change, because there are additional contributions to the relevant (CP violating as well as CP conserving) processes.

b. The two Standard Model sources, δ and θ may contribute in new ways, either because they appear in interactions of SUSY particles, or because of their effects through radiative corrections.

c. There may be two new sources of CP violation, ϕ_A and ϕ_B.

20.2. $\mathcal{D}_N$ FROM ϕ_A AND ϕ_B

The most stringent bound on the phases ϕ_A and ϕ_B comes from their contribution to the finite renormalization of $\bar{\theta}$ (through their contribution to quark mass matrices) [126]:

$$\delta\bar{\theta} \sim \frac{6\alpha_3}{4\pi}(\phi_A + \phi_B). \tag{20.14}$$

This leads to

$$|\phi_A + \phi_B| \lesssim 10^{-7}. \tag{20.15}$$

This suggests that, if ϕ_A and ϕ_B are different from zero, the theory should have an axion. In such a case, the most stringent bound comes from the direct contribution of ϕ_A and ϕ_B to quark EDMs (Fig. 11(a)) [126]:

$$\begin{aligned}\mathcal{D}_N \sim &\frac{e\alpha_3}{4\pi}\frac{|M_3 m_u \mu \langle H_d\rangle / \langle H_u\rangle|}{m_{3/2}^2 \max(m_{3/2}^2, |M_3|^2)}\arg(M_3^* B)\\ &+\frac{\alpha_3}{4\pi}\frac{|M_3 \xi_{U11}/\lambda_{U11}|}{m_{3/2}^2 \max(m_{3/2}^2, |M_3|^2)}\arg(M_3^* \xi_{U11}).\end{aligned} \tag{20.16}$$

Taking all supersymmetric parameters to equal $m_{3/2}$, this gives

$$\mathcal{D}_N \sim \left[\frac{100\ GeV}{m_{3/2}}\right]^2 \left[\frac{\arg(M_3^* B) + \arg(M_3^* \xi_U)}{10^{-3}}\right] 10^{-25}\ e\ \mathrm{cm}. \tag{20.17}$$

For $m_{3/2} \sim 100\ GeV$ this gives a bound of $\mathcal{O}(10^{-3})$ on ϕ_A and ϕ_B. For a higher SUSY breaking scale, $m_{3/2} \sim 1\ TeV$, the bound is milder, $\mathcal{O}(0.1)$. The SUSY contributions to the three gluon operator give similar bounds [131]. In any case, (20.17) implies that ϕ_A and ϕ_B have no interesting role in CP violation in neutral meson systems.

20.3. THE NEUTRAL K SYSTEM

There are several supersymmetric contributions to $M_{12}(K)$ (Fig. 11(b)):

1. The supersymmetric partners of the Standard Model box diagrams: For this not to exceed the experimental value of ΔM_K, near degeneracy among squarks is required (see (20.6)) and $\tilde{\xi}$ is required to be small.

2. The strong superbox diagrams with LR current structure: The experimental values of ΔM_K and ϵ require [126]

$$\begin{aligned}\mathrm{Re}\left(\frac{M_{\tilde{D}S}^2 M_{\tilde{S}D}^{*2}}{m_{3/2}^4}\right) &\lesssim 2\times 10^{-5}\frac{m_{3/2}^2}{m_W^2},\\ \mathrm{Im}\left(\frac{M_{\tilde{D}S}^2 M_{\tilde{S}D}^{*2}}{m_{3/2}^4}\right) &\lesssim 6\times 10^{-8}\frac{m_{3/2}^2}{m_W^2}.\end{aligned} \tag{20.18}$$

For $\tilde{\xi} = 0$ so that $\Delta\xi_D$ arises from RG scaling only, the contribution is negligible

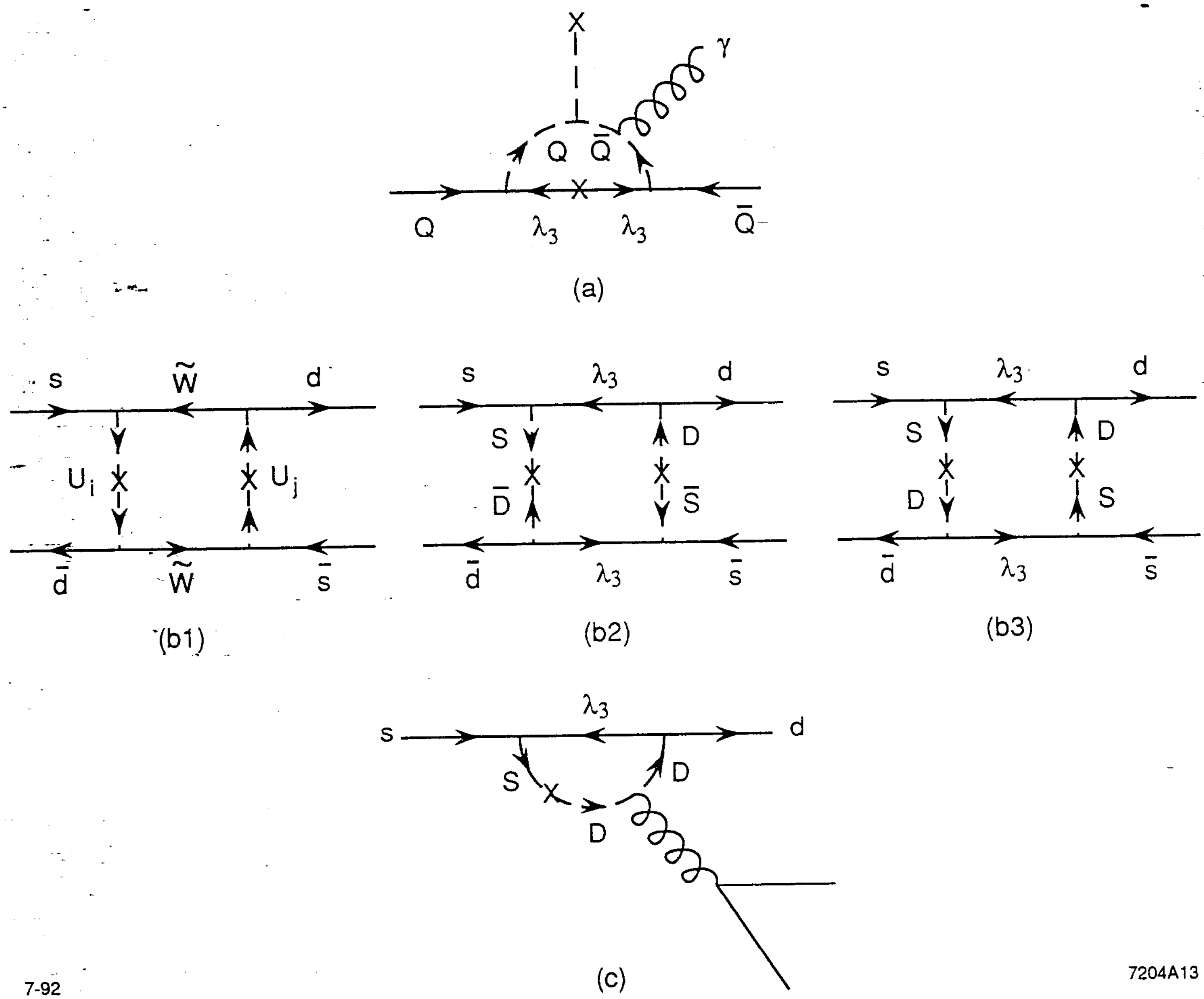

Figure 11. *CP* violating diagrams in the minimal SUSY framework. (a) A contribution to the EDM of the down quark. (b) Contributions to ϵ from box diagrams with (1) winos and squarks, (2) gluinos and squark doublets and singlets, and (3) gluinos and squark doublets. (c) A contribution to ϵ'.

because there is an extra power of small of quark mass,

$$\Delta\xi_D = \left(\frac{m_D}{m_W} V^\dagger \frac{m_U^2}{m_W^2} V\right). \tag{20.19}$$

Actually, the phase ϕ_A discussed in the previous section contributes through a similar diagram, and it is this extra suppression which renders its effect negligible.

3. The strong superbox diagrams with LL current structure: The contribution to ΔM_K is small. The contribution to ϵ is estimated to be

$$|\epsilon|_{\rm susy} = 300 \frac{m_t^4}{m_{3/2}^2 m_W^2} \frac{{\rm Im}[(V_{td}^* V_{ts})^2]}{2|V_{us}|^2}. \tag{20.20}$$

This leads roughly to

$$|J| \lesssim \frac{m_W^2 m_{3/2}^2}{m_t^4} 4 \times 10^{-4}, \tag{20.21}$$

which is weaker than direct bound on $|J|$. Conversely, the strong superbox diagram does not dominate over the Standard Model contribution to ϵ, but may be comparable for large m_t.

Supersymmetric penguin diagrams (Fig. 11(c)) give additional contributions to ϵ'/ϵ. While the GIM mechanism gives a logarithmic dependence on m_t for the Standard Model penguin, it gives a quadratic dependence on m_t for the SUSY penguin:

$$\frac{(\epsilon'/\epsilon)_{spen}}{(\epsilon'/\epsilon)_{peng}} \approx \frac{1}{5}\left(\frac{g_3}{g}\right)^2 \frac{(m_t/m_{3/2})^2}{\ln(m_t/m_c)^2}. \tag{20.22}$$

Again, for large m_t the SUSY contribution to ϵ'/ϵ may be large but will not change the order of magnitude estimate from the Standard Model.

20.4. CP ASYMMETRIES IN B DECAYS

The strong superbox diagrams contribute to $B-\bar{B}$ mixing. However, in the minimal SUSY models as defined above, the weak phases are exactly the CKM phases of the Standard Model. Consequently, $(q/p)_B = (M_{12}^*/M_{12})^{1/2}$ remains unchanged and the Standard Model predictions are not modified. This conclusion is independent of whether the SUSY contributions to $M_{12}(B)$ are large or not.

20.5. SUMMARY

The two new sources of CP violation that appear in minimal SUSY models, ϕ_A and ϕ_B, may saturate the upper bound on the EDM of the neutron even if the SUSY breaking scale is a few TeV. However, they have no impact on CP violation in neutral meson systems.

There are additional contributions to CP violation in the neutral K system which arise from δ_{KM} due to the existence of supersymmetric box diagrams that contribute to ϵ and supersymmetric penguin diagrams that contribute to ϵ'/ϵ. However, these contributions are at most comparable to the Standard Model contributions, and thus no significant constraints on the relevant parameters arise. If the SUSY breaking scale is above the electroweak breaking scale, then SUSY contributions to FCNC processes and, in particular, to CP violating processes are too small to have observable effects. The same seems to hold for models where electroweak breaking is induced radiatively, even if the SUSY breaking scale is not particularly large.

In the minimal SUSY model, the Standard Model predictions for CP asymmetries in B decays remain unchanged.

In extensions of the minimal SUSY models, such that $\tilde{\xi} \neq 0$, or where $M_{ab}^2 /\!\!\propto \delta_{ab}$, most of the above considerations do not hold and many different supersymmetric effects on CP violating observables may occur (see *e.g.* refs. [132, 133]). In ref. [130] it was shown that in non-minimal SUSY models there could be significant modifications of CP asymmetries in B decays. All asymmetries may have any value in the full range $[-1, 1]$.

21. Reducing the Number of Parameters

Schemes for Quark Mass Matrices

Various schemes for mass matrices predict relations among parameters of the quark sector. The hope is that, if these relations are experimentally verified, it will lead us to find symmetries that operate differently on different generations ("horizontal symmetries").

Some of these schemes are very powerful in their predictions for CP asymmetries in B decays. Instead of the Standard Model allowed range for the asymmetries depicted in Fig. 6, a much smaller range is allowed when the various mass and mixing parameters are related. Thus, the measurement of $a_{\psi K_S}$ and $a_{\pi\pi}$ will provide a stringent test for these schemes. In Fig. 12 we present the predictions of five schemes of quark parameters [71].

The Fritzsch scheme [134] assumes that the quark mass matrices have the following form:

$$M_d = \begin{pmatrix} 0 & a_d e^{i\phi_1} & 0 \\ a_d e^{-i\phi_1} & 0 & b_d e^{i\phi_2} \\ 0 & b_d e^{-i\phi_2} & c_d \end{pmatrix}, \quad M_u = \begin{pmatrix} 0 & a_u & 0 \\ a_u & 0 & b_u \\ 0 & b_u & c_u \end{pmatrix}. \tag{21.1}$$

The scheme by Giudice [135] assumes that, at the GUT scale, the fermion mass matrices have the following form:

$$M_d = \begin{pmatrix} 0 & f e^{i\phi} & 0 \\ f e^{-i\phi} & d & 2d \\ 0 & 2d & c \end{pmatrix}, \quad M_u = \begin{pmatrix} 0 & 0 & b \\ 0 & b & 0 \\ b & 0 & a \end{pmatrix}, \quad M_\ell = \begin{pmatrix} 0 & f & 0 \\ f & -3d & 2d \\ 0 & 2d & c \end{pmatrix}. \tag{21.2}$$

The scheme by Dimopoulos, Hall and Raby [136] assumes that, at the GUT scale,

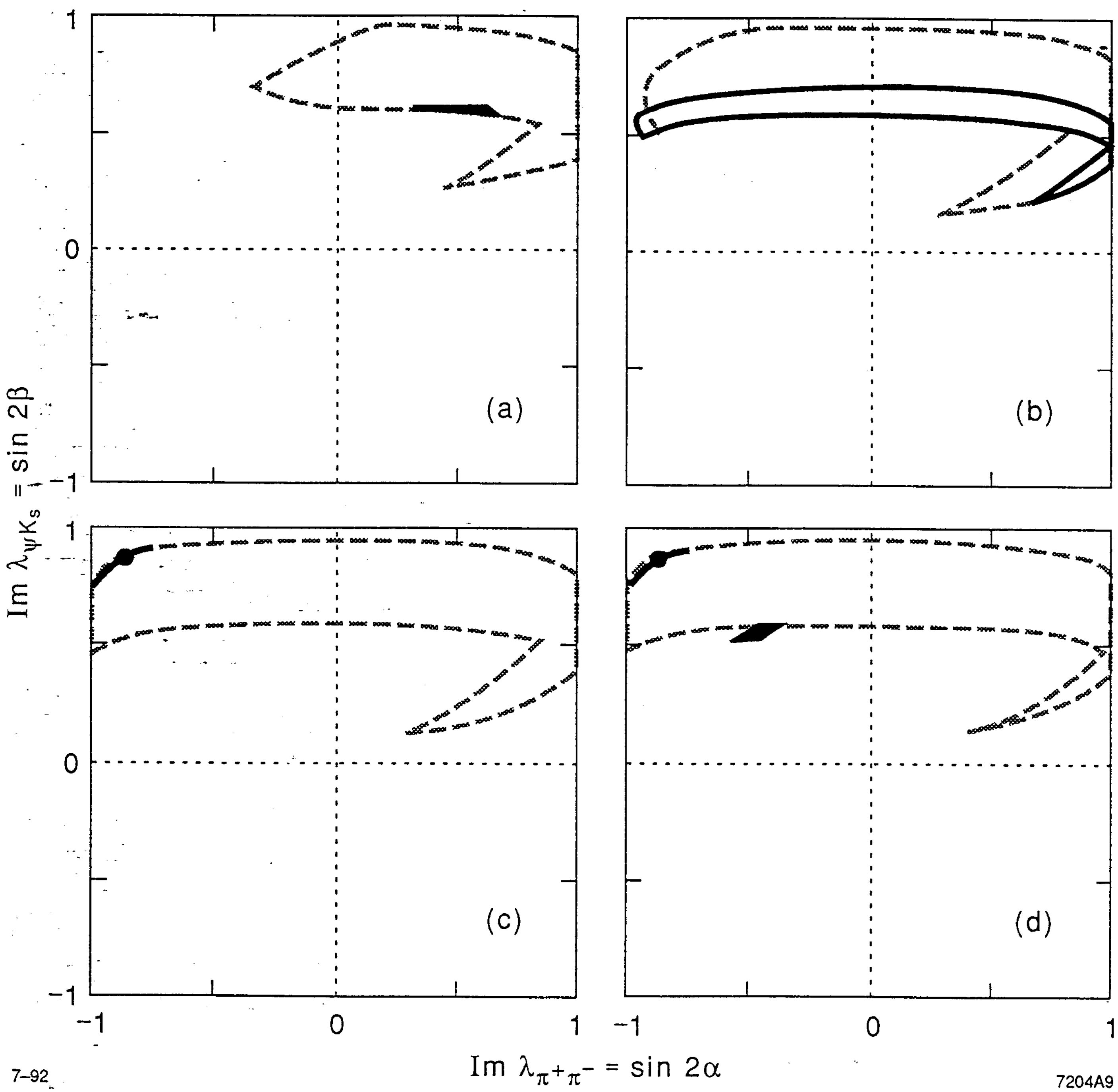

Figure 12. The predicted range for CP asymmetries in $B \to \psi K_S$ and $B \to \pi^+\pi^-$ in the Standard Model (dashed curves) and in various schemes for quark mass matrices. (a) $m_t = 90$ GeV. The black area gives the Fritzsch scheme prediction. (b) $m_t = 130$ GeV. The solid curve gives the Giudice scheme prediction. (c) $m_t = 160$ GeV. The solid curve gives the symmetric CKM prediction, while the dot is the prediction of Kielanowski's scheme. (d) $m_t = 185$ GeV. The solid curve gives the symmetric CKM prediction, the dot is the prediction of Kielanowski's scheme and the black area corresponds to the DHR scheme.

the fermion mass matrices have the following form:

$$M_d = \begin{pmatrix} 0 & fe^{i\phi} & 0 \\ fe^{-i\phi} & e & 0 \\ 0 & 0 & d \end{pmatrix}, \quad M_u = \begin{pmatrix} 0 & c & 0 \\ c & 0 & b \\ 0 & b & a \end{pmatrix}, \quad M_\ell = \begin{pmatrix} 0 & f & 0 \\ f & -3e & 0 \\ 0 & 0 & d \end{pmatrix}. \tag{21.3}$$

The "symmetric CKM" ansatz [137] assumes for the elements of the CKM matrix

$$|V_{ij}| = |V_{ji}|. \tag{21.4}$$

The ansatz by Kielanowski [138] assumes, in addition to (21.4),

$$|V_{23}| = \frac{|V_{12}V_{13}|}{|V_{12}|^2 + |V_{13}|^2}(3 - |V_{12}|^2 - |V_{13}|^2)^{1/2}. \tag{21.5}$$

Taking into account the expected experimental accuracy in a B factory ($\mathcal{O}(0.05)$ in $a_{\psi K_S}$ and $\mathcal{O}(0.10)$ in $a_{\pi\pi}$), we conclude that each of these schemes may be clearly excluded when the asymmetries are measured.

22. Conclusions

Most extensions of the Standard Model suggest that there are many new sources of CP violation, beyond the single phase of the CKM matrix. Such additional phases have two typical consequences:

(i) If the phases occur in flavor changing couplins to quarks, the very strong Standard Model relation between CP violation in the K system and in the B system is lost. Instead of the narrow range allowed by the Standard Model for CP asymmetries in neutral B decays, the whole possible range may be allowed in such extensions.

(ii) If the phases occur in flavor-diagonal couplings, the value of the electric dipole moment of the neutron is orders of magnitudes above its Standard Model value. In many models the experimental bound on $\mathcal{D}_n$ is almost saturated. Similarly, the value of $\mathcal{D}_e$ may be very close to the experimental bound.

The conclusion is that constructing a B factory to measure the CP asymmetries in neutral B decays, and the experimental efforts to improve the sensitivity to the EDMs of the neutron and the electron may be well rewarded: it is not unlikely that new physics will be discovered in these experiments.

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