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SEARCH FOR GLUONS IN e^+e^- ANNIHILATION

John ELLIS, Mary K. GAILLARD * and Graham G. ROSS
CERN, Geneva

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We study the deviations to be expected at high energies from the recently observed two-jet structure of hadronic final states in e^+e^- annihilation. Motivated by the approximate validity of the naïve parton model and by asymptotic freedom, we suggest that hard gluon bremsstrahlung may be the dominant source of hadrons with large momenta transverse to the main jet axes. This process should give rise to three-jet final states. These may be observable at the highest SPEAR or DORIS energies, and should be important at the higher PETRA or PEP energies.

1. Introduction

One of the most beautiful experimental discoveries in e^+e^- annihilation during the past year has been the evidence [1] for two-jet structures in continuum hadronic final states at centre-of-mass energies $Q \geqslant 6$ GeV. These observations seem to confirm predictions [2] made using parton models, both as to the existence of jets and their angular distribution. The predictions were based on the simple graph of fig. 1 with fundamental spin-$\frac{1}{2}$ (quark) constituents, which were supposed to metamorphose into hadrons with small momenta transverse to the quarks' directions of motion. Indeed the value of the average $p_T \sim 300$ MeV suggested by the experimental data is similar to that found for particles generated in hadronic collisions.

If this state of affairs persists into PETRA and PEP energies, then the hadrons in the two jets will be so highly collimated that separating and distinguishing them experimentally will be very difficult. It is, however, expected that $\langle p_T \rangle$ will grow to some extent as $\sqrt{s}$ increases. For example, Kogut and Susskind [3] have predicted $\langle p_T \rangle \propto \sqrt{s}$ at high energies as a consequence of hard fundamental processes in their scale invariant parton model. Also, we know there is a large and increasing cross section for producing large p_T hadrons in pp collisions.

However, there are dynamical differences between pp and e^+e^- collisions which lead us to suspect that different mechanisms may generate large p_T hadrons in the

* And Laboratoire de Physique Théorique et Particules Elémentaires, Laboratoire associé au CNRS, Orsay.

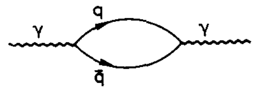

Fig. 1. The simple parton model for $e^+e^- \to \gamma \to$ hadrons which gives two jets.

two cases, and that these differences may be reflected in the structure of the final states. Thus hard scattering models [4] regard pp collisions as taking place between collimated beams of constituents [4,5] (partons ?) which sometimes have hard scatterings (possibly scale invariant ?).

In such events the hadronic final state should [4–6] contain two large p_T jets from the metamorphoses of the hard scattering constituents, as well as fragmentation jets of the target and projectile aligned along the collision axis. This picture, indicated in fig. 2, is indeed supported by elegant recent observations [7] at the CERN-ISR. By contrast, e^+e^- collisions in the one-photon exchange approximation are visualized (fig. 1) as having essentially free parton constituents flying apart, in which case there is no obvious collision mechanism for generating large p_T structures in the final state.

However, parton constituents may not be completely free at high momenta (short distances). All field theory models require them to have some interactions, if only with uncharged bosonic gluons. None of these interactions vanishes identically at large momenta, though a non-Abelian gauge vector gluon coupling can be asymptotically free: $g_V^2 \sim 1/\ln Q^2$ as $Q^2 \to \infty$. No direct experimental evidence yet exists for gluons, except possibly the fact that not all the nucleon's momentum is carried by known quark constituents [8]. Similarly, there is no direct evidence for asymptotic freedom, though there may be some deviations [9] from scaling in deep inelastic scattering at high Q^2. Also, some of the difference [10] between

$$R \equiv \frac{\sigma(e^+e^- \to \gamma \to \text{hadrons})}{\sigma(e^+e^- \to \gamma \to \mu^+\mu^-)}$$

and the value $\Sigma_{\text{quarks } a}\, Q_a^2$ suggested by the graph of fig. 1 may be due to gluonic corrections, given dominantly by fig. 3 in an asymptotically free theory. For consistency with these constraints within asymptotically free perturbation theory, fashion sets the non-Abelian coupling $\alpha_V(Q^2) \equiv g_V^2/4\pi \sim 0.2$ to 1 at $Q^2 \sim 10$.

In such a framework, Kogut and Susskind [3] pointed out that the diagrams of fig.

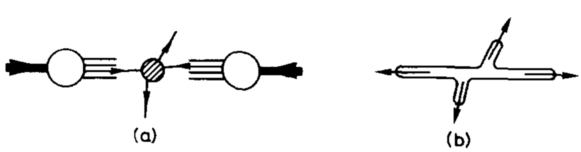

Fig. 2. (a) The hard scattering model for large p_T hadronic collisions, and (b) the momentum space structure of the final state hadrons.

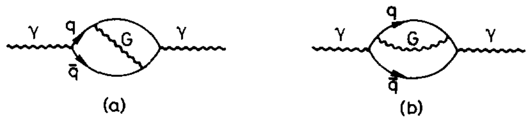

Fig. 3. Leading corrections to $e^+e^- \to \gamma \to$ hadrons in an asymptotically free theory.

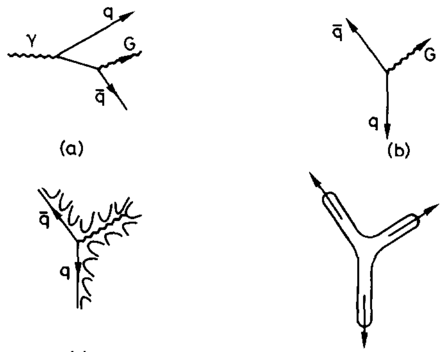

Fig. 4. (a) The graph for hard gluon bremsstrahlung; (b) in momentum space; (c) metamorphosis into hadrons, and (d) the momentum space structure of the final state.

3 may be expected to show up in hadronic final states, with the dominant supplements to two-jet events coming from hard gluon bremsstrahlung generating hadrons at high p_T. In the usual picture of constituents' metamorphoses into final state hadrons, these should be identifiable as three-jet events if the momentum transfers are high enough, as suggested by fig. 4. A striking feature of this mechanism is that there is no "quasi-local" compensation of large p_T due to another jet on the opposite side of the parton-antiparton axis, as predicted [4–6] and found [7] in hadronic collisions. Instead the p_T compensation is "non-local", with both the parton and antiparton jet-axes being bent around to compensate the hard bremsstrahlung *.

In this paper we calculate the production of large p_T hadrons by both scalar and vector gluon bremsstrahlung. Using a fashionable value for the gluon coupling constant, we estimate the cross sections for large p_T hadron production, and the angular distributions and observability of three-jet events, for different c.m. energies. We have the scaling law:

$$\frac{1}{\sigma_{\text{total}}} \frac{d\sigma}{dx_T} \sim f(x_T)\,, \qquad x_T = \frac{2P_T}{Q}\,,$$

* This is reminiscent of the non-local compensation of other quantum numbers, such as strangeness, expected to occur in deep inelastic processes.

to within possible logarithms. We find deviations from the $\exp(-6p_T)$ distribution found in hadronic collisions which are negligible for $Q \sim 6$ GeV, small but quite possibly detectable for $Q \sim 8$ GeV, and very substantial for PEP and PETRA energies $10\ \mathrm{GeV} \leqslant Q \leqslant 40$ GeV.

The organization of the paper is as follows: in sect. 2 we discuss kinematics and calculate the basic hard gluon bremsstrahlung cross sections. In sect. 3 we discuss the metamorphosis of gluon into hadrons, and calculate hadronic cross sections and total event transverse momenta relative to different axes chosen for analysis. In sect. 4 we discuss our conclusions.

2. Kinematics and bremsstrahlung calculations

We will be calculating the process $e^+e^- \to \gamma \to q\bar{q}G$ of fig. 5, where q and $\bar{q}$ are quark and antiquark, and G is a gluon which is either vector or scalar, and possibly coloured. The three final state "particles" will lie in a plane, which we specify as indicated in fig. 6. The plane is defined to make an angle θ to the e^- momentum q_1, and to be oriented with an azimuthal angle ϕ. We name the q, $\bar{q}$ and G momenta $p_{1,2,3}$, respectively, and suppose them to make angles ϕ_i, $i = 1, 2, 3$ with the projection of q_1 in the final state plane. The variables (θ, ϕ, ϕ_i) specify the kinematics given the incoming $e^{\pm}$ beam energies E, in terms of which the virtual intermediate photon has $Q^2 = 4E^2$. If we call the final state energies E_i, $i = 1, 2, 3$, and suppose quarks and

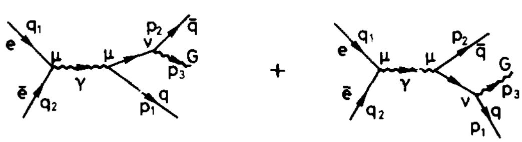

Fig. 5. Graphs and notation for $e^+e^- \to \gamma \to q\bar{q}G$.

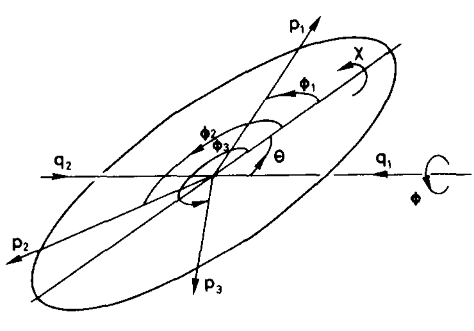

Fig. 6. Kinematical definitions.

gluons to be effectively massless, which should be a good approximation at the high energies ($Q^2 \gtrsim 36$) we will be studying, then:

$$q_1 \cdot p_i = EE_i(1 - \cos\theta \cos\phi_i) ,$$
$$q_2 \cdot p_i = EE_i(1 + \cos\theta \cos\phi_i) . \tag{2.1}$$

Energy-momentum conservation tells us that

$$\sum_{i=1}^{3} E_i = 2E ,$$
$$\sum_{i=1}^{3} E_i \cos(\phi_i - \phi_0) = 0 , \tag{2.2}$$

for any angle ϕ_0. Therefore

$$E_1 = \frac{2E \sin(\phi_3 - \phi_2)}{\sin(\phi_3 - \phi_2) + \sin(\phi_1 - \phi_3) + \sin(\phi_2 - \phi_1)} \tag{2.3}$$

and similarly for $E_{2,3}$ by cyclic permutation. It will sometimes be useful to exploit the variables

$$s_{ij} = (p_i + p_j)^2$$
$$= 4E(E - E_k) , \tag{2.4}$$

where $k \neq i \neq j$. Introducing the conventional scaling variables

$$x_i = \frac{E_i}{E} = \frac{2E_i}{Q} , \tag{2.5}$$

we have

$$s_{ij} = Q^2(1 - x_k) . \tag{2.6}$$

Because the final state "particles" are essentially massless

$$Q^2 = s_{23} + s_{31} + s_{12} . \tag{2.7}$$

The process can be specified in terms of θ, ϕ, two of the s_{ij}, and the orientation χ of an arbitrary axis in the plane. In terms of these variables the Lorentz-invariant phase-space factor

$$\frac{1}{(2\pi)^5} \delta^4(q_1 + q_2 - p_1 - p_2 - p_3) \frac{d^3p_1 d^3p_2 d^3p_3}{8E_1E_2E_3}$$
$$= \frac{1}{(2\pi)^5} \delta^4(q - p_1 - p_2 - p_3) \frac{d^4q}{32Q^2} ds_{12} ds_{23} d\chi d\cos\theta \, d\phi , \tag{2.8}$$

where $q = q_1 + q_2$, $q^2 = Q^2$.

In terms of the variables introduced above, the Feynman graphs of fig. 5 yield a cross section

$$d\sigma^V = \frac{1}{(2\pi)^5} \delta^4(q_1 + q_2 - p_1 - p_2 - p_3) |T_V|^2 \frac{d^3p_1 d^3p_2 d^3p_3}{64E^2E_1E_2E_3}, \tag{2.9}$$

where

$$T_V = \frac{e^2 g_V}{Q^4} \bar{v}(q_2)\gamma^\mu u(q_1) \left\{ \bar{u}(p_1)\gamma_\nu \frac{(\not{p}_1 + \not{p}_3)}{(p_1+p_3)^2} \gamma_\mu v(p_2) \right.$$
$$\left. - \bar{u}(p_1)\gamma_\mu \frac{(\not{p}_2 + \not{p}_3)}{(p_2+p_3)^2} \gamma_\nu v(p_2) \right\} \epsilon^\nu_\lambda \tag{2.10}$$

is the amplitude for emitting a vector gluon in a polarization state λ from unit charge quarks *via* a $\bar{q}qG$ coupling constant g_V. Averaging over $e^\pm$ polarizations and summing over final state polarizations, we find

$$\sum \overline{|T_V|^2} = \frac{e^4 g_V^2}{4Q^4} L^{\mu\nu} H^V_{\mu\nu}, \tag{2.11}$$

where the lepton trace

$$L^{\mu\nu} = 4(q_2^\mu q_1^\nu + q_1^\mu q_2^\nu - q_1 \cdot q_2 g^{\mu\nu})$$
$$\equiv 4\{q_2, q_1\}^{\mu\nu}, \tag{2.12}$$

and the hadron trace

$$\tfrac{1}{4} H^V_{\mu\nu} = \frac{1}{(p_1 \cdot p_3)} [\{p_2, p_3\}_{\mu\nu} - \{p_1, p_1\}_{\mu\nu} + \{p_1, p_2\}_{\mu\nu}]$$
$$+ \frac{1}{(p_2 \cdot p_3)} [\{p_1, p_3\}_{\mu\nu} - \{p_2, p_2\}_{\mu\nu} + \{p_1, p_2\}_{\mu\nu}]$$
$$+ \frac{(p_1 \cdot p_2)}{(p_1 \cdot p_3)(p_2 \cdot p_3)} [2\{p_1, p_2\}_{\mu\nu} + \{p_1, p_3\}_{\mu\nu} + \{p_2, p_3\}_{\mu\nu}] \tag{2.13}$$

in the notation introduced in eq. (2.12). Combining (2.12) and (2.13), we find

$$\sum \overline{|T_V|^2} = \frac{e^4 g_V^2 Q^2}{2(p_1 \cdot p_3)(p_2 \cdot p_3)} \{(1 - x_1)[x_1 x_2 (1 - \cos^2\theta \cos\phi_1 \cos\phi_2)$$
$$- x_1^2 (1 - \cos^2\theta \cos^2\phi_1) + x_2 x_3 (1 - \cos^2\theta \cos\phi_2 \cos\phi_3)]$$

$$+ (1-x_2)\,[x_1x_2(1-\cos^2\theta\,\cos\phi_1\,\cos\phi_2) - x_2^2(1-\cos^2\theta\,\cos^2\phi_2)$$

$$+ x_1x_3(1-\cos^2\theta\,\cos\phi_1\,\cos\phi_3)]$$

$$+ (1-x_3)\,[2x_1x_2(1-\cos^2\theta\,\cos\phi_1\,\cos\phi_2)$$

$$+ x_1x_3(1-\cos^2\theta\,\cos\phi_1\,\cos\phi_3) + x_2x_3(1-\cos^2\theta\,\cos\phi_3)]\} \,. \tag{2.14}$$

Combining eqs. (2.9) and (2.14), and using the formula (2.8) for phase space gives the fivefold differential cross section for $e^+e^- \to \gamma \to q\bar{q}G$ for unpolarized beams.

In practice, $e^\pm$ beams are polarized [1] at SPEAR energies, and it should be useful to our experimental colleagues to have the cross sections for polarized beams. This can be given using a simple recipe of Bjorken [11]. Let $d\sigma/d\Gamma$ be the unpolarized cross section calculated above, let $(d\sigma/d\Gamma)_x$ be the cross section into the same laboratory configuration which would have been obtained from the analogue of (2.14) if the e^- been had been coming from the direction $\phi = 0, \theta = \frac{1}{2}\pi$, and let $(d\sigma/d\Gamma)_y$ be the corresponding cross section if the e^- beam had been coming from the direction $\phi = \frac{1}{2}\pi, \theta = \frac{1}{2}\pi$. Then

$$\left.\frac{d\sigma}{d\Gamma}\right|_{\substack{\text{polarized}\\ \text{beams}}} = \frac{d\sigma}{d\Gamma} + \left|P_+P_-\right| \left\{\left(\frac{d\sigma}{d\Gamma}\right)_x - \left(\frac{d\sigma}{d\Gamma}\right)_y\right\}, \tag{2.15}$$

where $P_\pm$ are the $e^\pm$ beam polarizations, and the sign corresponds to that expected and found in practice.

The fivefold differential cross section is probably too differential to be useful. Integration over the azimuthal angle ϕ gives a trivial factor of 2π, and simplifying (2.14) by means of eqs. (2.2), we get (introducing $\alpha_V \equiv g_V^2/4\pi$ and using unit charge quarks):

$$d\sigma^V = \frac{\alpha^2}{8\pi}\frac{\alpha_V}{Q^2}\,(x_1^2 + x_2^2 + \cos^2\theta\,(x_2^2\cos^2\phi_2 + x_1^2\cos^2\phi_1))$$
$$\times \frac{ds_{13}}{s_{13}}\frac{ds_{23}}{s_{23}}\,d(\cos\theta)\,d\chi\,. \tag{2.16}$$

If we then integrate over the Euler angle χ, we find

$$d\sigma^V = \frac{\alpha^2}{8}\frac{\alpha_V}{Q^2}\,(x_1^2 + x_2^2)(2 + \cos^2\theta)\,\frac{ds_{13}}{s_{13}}\frac{ds_{23}}{s_{23}}\,d(\cos\theta) \tag{2.17}$$

for the distribution of the angle between the event plane and the beam axes. Finally, if we integrate over the angle θ we obtain

$$\frac{d^2\sigma^V}{ds_{13}ds_{23}} = \frac{7}{12Q^2}\,\alpha^2\alpha_V\,\frac{x_1^2 + x_2^2}{s_{13}s_{23}} \tag{2.18}$$

for $e^+e^- \to \gamma \to q\bar{q}G$ with one quark of unit charge.

In colour gauge theories, the coupling is conventionally defined by the interaction term $g_V \bar{q}(\frac{1}{2}\lambda_i)\gamma_\mu q G_i^\mu$. Then, summing over the colours of the final state particles yields an additional factor 4 relative to the abelian gluon case calculated in equations (2.16)–(2.18). Assuming colour and introducing the point-like cross section

$$\sigma_{pt}(e^+e^- \to \gamma \to \text{hadrons}) \equiv 3 \sum_{\substack{\text{quark} \\ \text{flavours } a}} Q_a^2 \times \sigma(e^+e^- \to \gamma \to \mu^+\mu^-)\,,$$

we find

$$\frac{1}{\sigma_{pt}} \frac{d^2\sigma^V}{ds_{13}ds_{23}} = \frac{7}{12\pi}\alpha_V \frac{x_1^2 + x_2^2}{s_{13}s_{23}}\,. \tag{2.19}$$

The corresponding calculations for (coloured) scalar gluons are equally straightforward. They yield

$$\begin{aligned}\tfrac{1}{4}H_{\mu\nu}^S &= \frac{1}{2(p_1\cdot p_3)}\left[\{p_2,p_3\}_{\mu\nu} + \{p_1,p_3\}_{\mu\nu}\right] \\ &+ \frac{1}{2(p_2\cdot p_3)}\left[\{p_1,p_3\}_{\mu\nu} + \{p_2,p_3\}_{\mu\nu}\right] \\ &- \frac{(p_1\cdot p_2)}{2(p_1\cdot p_3)(p_2\cdot p_3)}\{p_3,p_3\}_{\mu\nu}\end{aligned} \tag{2.20}$$

to replace eq. (2.13). Then

$$\begin{aligned}\sum \overline{|T_S|^2} = \frac{e^4 g_S^2 Q^2}{4(p_1\cdot p_3)(p_2\cdot p_3)} &\{(2 - x_1 - x_2)\,[x_1x_3(1-\cos^2\theta\cos\phi_1\cos\phi_3) \\ &+ x_2x_3(1-\cos^2\theta\cos\phi_2\cos\phi_3)] - (1-x_3)x_3^2(1-\cos^2\theta\cos^2\phi_3)\}\end{aligned} \tag{2.21}$$

with g_S the $\bar{q}q$ scalar gluon coupling constant to replace eq. (2.14). Integrating over all the angles χ, θ and ϕ gives

$$\frac{d^2\sigma^S}{ds_{13}ds_{23}} = \frac{7}{24Q^2}\alpha^2\alpha_S \frac{x_3^2}{s_{13}s_{23}}\,, \tag{2.22}$$

using $\alpha_S \equiv g_S^2/4\pi$, and finally

$$\frac{1}{\sigma_{pt}} \frac{d^2\sigma^S}{ds_{13}d_{23}} = \frac{7}{24\pi}\alpha_S \frac{x_3^2}{s_{13}s_{23}} \tag{2.23}$$

for (coloured) scalar gluons.

The cross sections (2.19) and (2.23) are peaked towards low quark-gluon invariant masses, and indeed have infra-red divergences. These will be cancelled [12] by the infra-red divergences in virtual gluon corrections to the process $e^+e^- \to \gamma \to \bar{q}q$, and also coming from the graphs of fig. 3. The infra-red divergences can be removed by any reasonable cut-off procedure. In our case, we believe the description of hadronic processes in terms of fundamental quarks and gluons is only sensible when momentum (transfers)$^2 \geqslant$ some Q_0^2 are involved. The apparent plateau in the $e^+e^- \to \gamma \to$ hadrons to $e^+e^- \to \gamma \to \mu^+\mu^-$ ratio of ~2.5 for $Q \geqslant 3$ GeV suggests $Q_0^2 \sim 10$ GeV^2 may be reasonable. If Q_0^2 can be used as an approximate cut-off in s_{13} and s_{23} * for the cross sections (2.19) and (2.23) to apply, then we have

$$\frac{\sigma^V}{\sigma_{pt}} = \frac{28}{3\pi}\alpha_V\, I^V\left(\frac{Q_0^2}{Q^2}\right), \tag{2.24}$$

where

$$I^V(\epsilon) = \tfrac{1}{16}\int_{\epsilon}^{1-2\epsilon}\frac{dy}{y}\int_{\epsilon}^{1-\epsilon-y}\frac{dx}{x}\,[(1-x)^2 + (1-y)^2]\,. \tag{2.25}$$

Integrating eq. (2.25) we find that for small ϵ:

$$I^V(\epsilon) \simeq \tfrac{1}{16}\,[2|\ln\epsilon|^2 - 3|\ln\epsilon| + \tfrac{5}{2} - \tfrac{1}{3}\pi^2 + 2\epsilon|\ln\epsilon| + 4\epsilon + \ldots]\,. \tag{2.26}$$

Correspondingly, we have a scalar gluon cross section resembling (2.24) with α_V replaced by α_S and I^V by

$$I^S(\epsilon) = \tfrac{1}{32}\int_{\epsilon}^{1-2\epsilon}\frac{dy}{y}\int_{\epsilon}^{1-\epsilon-y}\frac{dx}{x}\,(x+y)^2$$

$$= \tfrac{1}{32}\,\left[(1-2\epsilon)\ln\left(\frac{1-2\epsilon}{\epsilon}\right) - \tfrac{1}{2} + \tfrac{9}{2}\epsilon^2\right]\,. \tag{2.27}$$

If we insert $Q_0^2 \approx 10$ GeV^2 into eq. (2.24), and use $Q = 8$ GeV, we find $I^V \approx 0.1$. For $Q = 16$ or 32 GeV, I^V becomes $0(1)$: this reflects the fact that for small ϵ asymptotically free perturbation theory in α_V breaks down. If we keep the cut-off ϵ fixed (thus keeping within the strict domain of applicability of asymptotically free perturbation theory) and let $Q^2 \to \infty$, then I^V remains fixed, and we expect the cross-section ratio (2.24) to decrease as $1/\ln Q^2$. As a fixed cut-off in ϵ is a fixed cut-off in the angular variables ϕ_i, this expectation agrees with the conclusions of Sterman [13]. He argues on the basis of other massless field theories that in non-Abelian gauge theories all the $e^+e^- \to \gamma \to$ hadrons cross section should look like two jets

* This amounts to requiring that the electromagnetic and gluonic currents act close together in configuration space.

asymptotically, where he defines jets as having fast particles emerging within a finite angular cone asymptotically. Remembering from sect. 1 that fashion sets $\alpha_V(Q^2 \approx 10) \approx 0.2$ to 1, we see that at $Q \approx 8$ GeV, around 10% of the total continuum hadronic cross section could arise from hard gluon bremsstrahlung. The next questions are the phenomenological ones of how the bremsstrahlung process should be reflected in the hadronic final state, and whether its effects are experimentally detectable.

3. Phenomenological consequences

The recently observed [1] two-jet structure in e^+e^- annihilation only became clearly visible for $Q \geqslant 6$ GeV: at low energies there was not enough phase space for jets to be distinguished from baloons. Likewise, higher energies will probably be necessary to see any deviations from this structure due to hard gluon bremsstrahlung. Notice, however, that the power-law peaking (2.18), (2.22) of the cross section is less marked than the exponential behaviour typical of hadronic processes, such as e^{bt} in quasi-two-body reactions and e^{-Bp_T} in inclusive reactions, so that its effects should show up eventually.

To the extent that the single hard bremsstrahlung process dominates corrections to two-jet events, we expect the final state to reflect its fundamental three-particle nature, and be coplanar. Thus the two-jet cigars should have corrections resembling asymmetric frisbees (see fig. 7). Note also that the bremsstrahlung plane has a different angular distribution $\propto (2 + \cos^2\theta)$ [eq. (2.17)], compared with the expected spin-$\frac{1}{2}$ two-jet distribution $\propto (1 + \cos^2\theta)$. Following a suggestion of Bjorken and Brodsky [14], experimentalists [1] have been constructing a sort of inertia tensor in momentum space:

$$I_{ij} = \sum_a (\delta_{ij}|p^a|^2 - p_i^a p_j^a) , \tag{3.1}$$

where the sum over a runs over the charged particles in a given event, and (i, j) are 3-vector indices. The tensor I_{ij} is then diagonalized event-by-event, and the axis

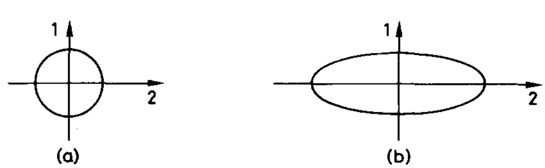

Fig. 7 (a) The transverse section of a two-jet cigar, and (b) the oblate transverse section of a frisbee shape.

found which minimizes $\Sigma_a |p_T^a|^2$. The sphericity

$$S = \frac{3 \sum_a |p_T^a|^2}{2 \sum_a |p^a|^2} \tag{3.2}$$

is then calculated and found to resemble that expected for two-jet structures with $\langle |p_T^a| \rangle \sim 300$ MeV. In terms of the eigenvalues $\lambda_1 \geqslant \lambda_2 \geqslant \lambda_3$ of I_{ij}

$$S = \frac{3\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} \,. \tag{3.3}$$

We would expect the growing oblateness of the hadronic cigar indicated in fig. 7 to be reflected in a tendency for

$$\sum_a p_2^{a2} > \sum_a p_1^{a2}$$

systematically. This would be reflected in larger values for $(\lambda_1 - \lambda_2)$ than expected in the axisymmetric jet model.

To calculate the magnitude of such effects in our hard gluon bremsstrahlung model, we first impose the cut-off $s_{13}, s_{23} \geqslant Q_0^2$ discussed in sect. 2 ★. We then postulate identical fragmentation functions $f(x)$ for a charged hadron with momentum fraction x to be produced in the metamorphosis of either a quark or a gluon into hadrons. We then find the axis in the final-state plane which minimizes

$$\sum_a p_T^{a2} \approx \sum_{i=1}^{3} \int_0^1 x^2 p_{T_i}^2 \, f(x) \mathrm{d}x \tag{3.4a}$$

$$= \left(\sum_{i=1}^{3} p_{T_i}^2 \right) \int_0^1 x^2 f(x) \mathrm{d}x \,, \tag{3.4b}$$

where the axis angle is α relative to the p_1 direction, so that

$$p_{T_1}^2 = E_1^2 \sin^2 \alpha \,,$$

$$p_{T_2}^2 = E_2^2 \sin^2 (\alpha + \phi_2 - \phi_1) \,,$$

$$p_{T_3}^2 = E_3^2 \sin^2 (\alpha + \phi_3 - \phi_1) \,, \tag{3.5}$$

★ Because of the logarithms in eqs. (2.26) and (2.27), this procedure is dangerous at higher values of Q. We find, however, that the cross sections at high p_T are largely independent of this procedure, as discussed later in this section.

and we have neglected hadron-hadron correlations in approximating $\Sigma_a\, p_T^2$ by the expression (3.4a). In terms of the E_i and ϕ_i:

$$\tan 2\alpha = \frac{E_2^2 \sin 2(\phi_1 - \phi_2) + E_3^2 \sin 2(\phi_1 - \phi_3)}{E_1^2 + E_2^2 \cos 2(\phi_1 - \phi_2) + E_3^2 \cos 2(\phi_1 - \phi_3)} \,. \tag{3.6}$$

To calculate the distribution of $\Sigma_a\, p_T^2$ we need to assume a form for $f(x)$. To get a fashionable logarithmic multiplicity within a jet $f(x) \sim 1/x$ as $x \to 0$. In fig. 8 we have compared

$$xf(x) = 2.2\,(1-x)^3 + 0.25\,(1-x) \tag{3.7}$$

with the charged particle spectra from SPEAR. Formula (3.7) is supposed to be a

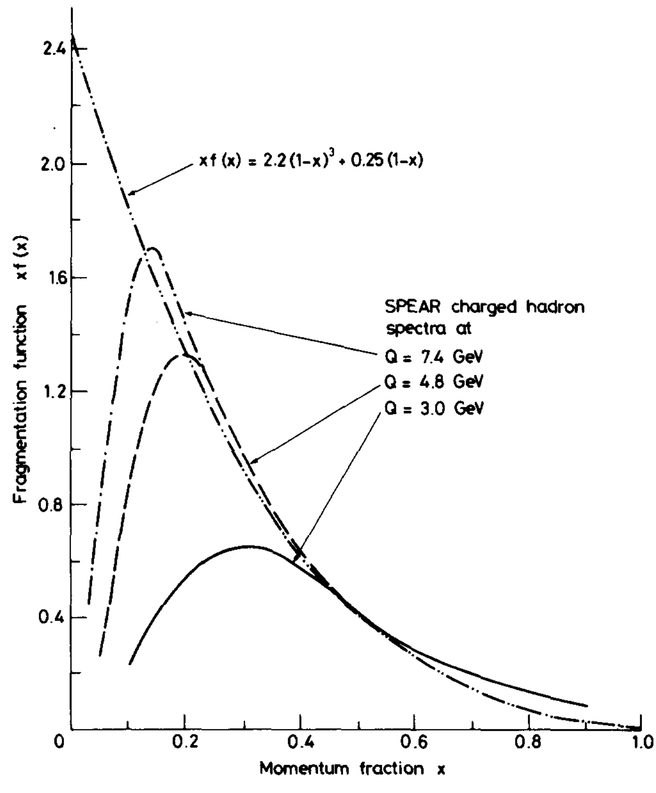

Fig. 8. The model (3.7) for $f(x)$ compared with SPEAR data. The data have been reduced by a factor 10 to allow for $R = 5$ and two jets at high energies.

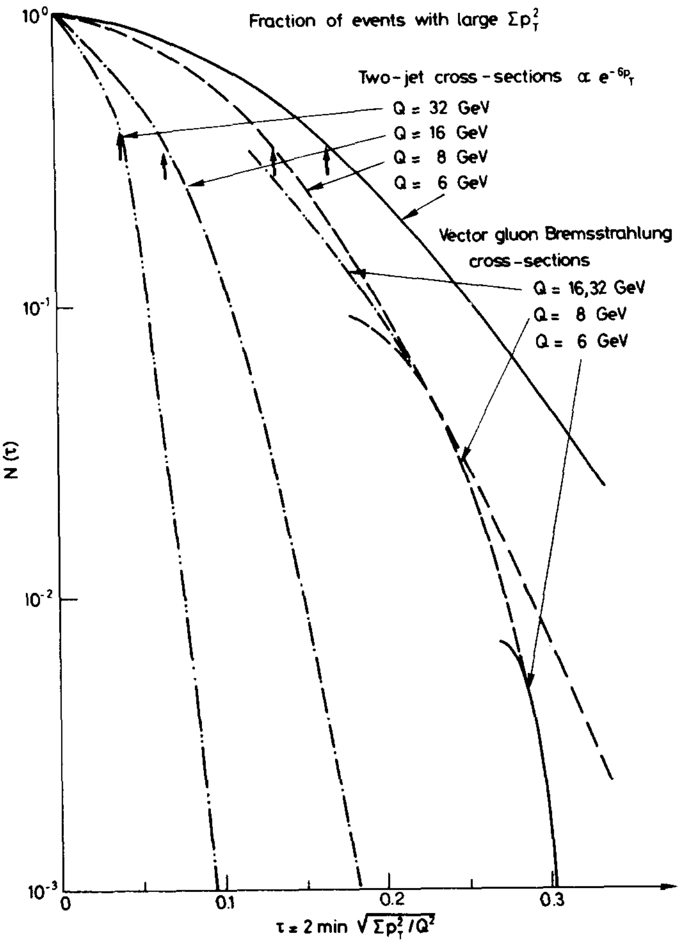

Fig. 9. The number $N(\tau)$ of events with large total transverse momentum, with the cut-offs discussed in the text. The vertical arrows indicate the averages of Σp_T^2.

qualitative representation rather than a best fit. For the model (3.7)

$$\int_0^1 x^2 f(x)\mathrm{d}x \simeq 0.15$$

and in fig. 9, we have plotted the cross section

$$N(\tau) \equiv \frac{\sigma\left(\sqrt{\sum_a p_T^{a2}/Q^2} \geqslant \tau\right)}{\sigma_{\mathrm{pt}}} \tag{3.8}$$

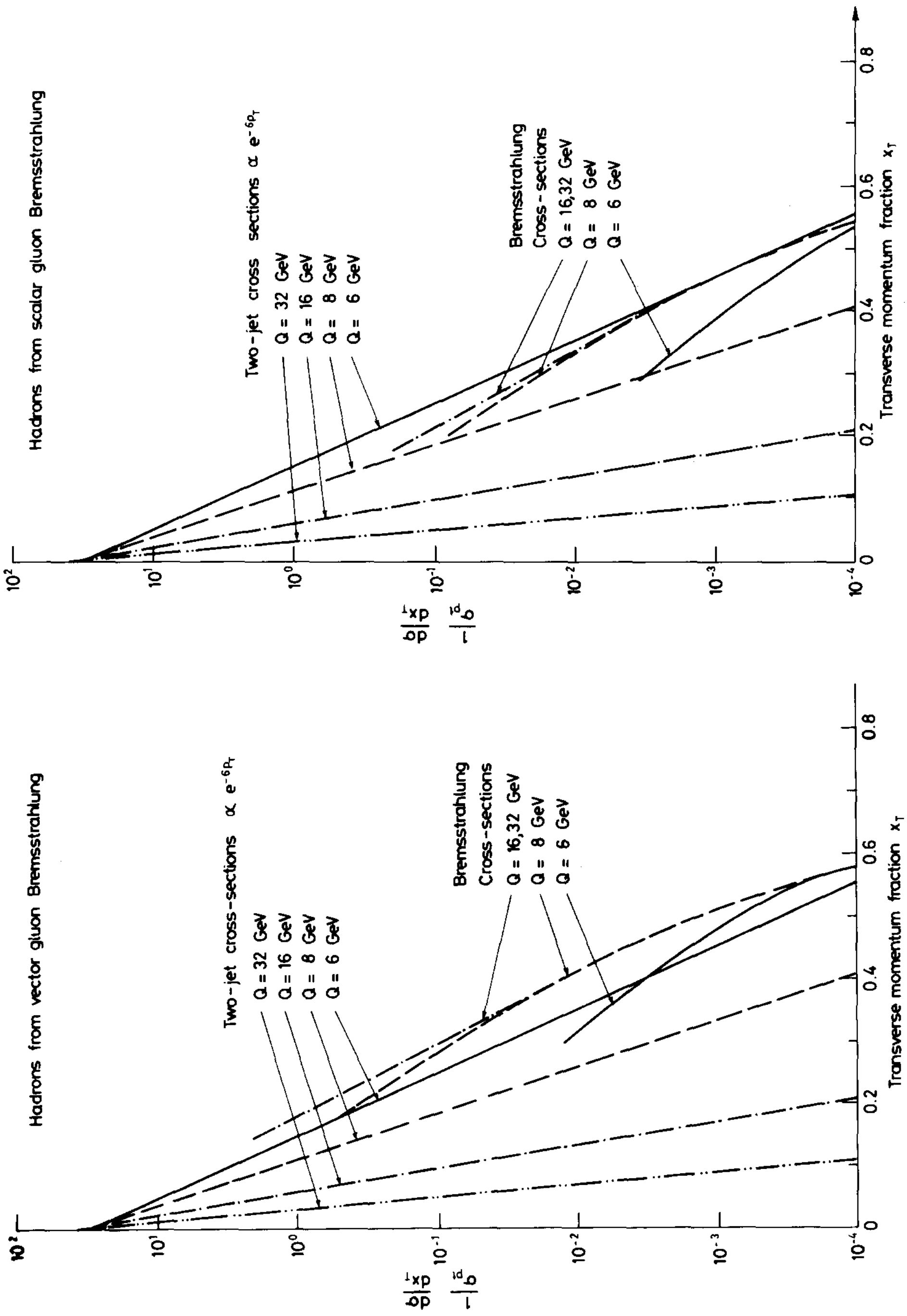

Fig. 10. Transverse momentum cross sections relative to the α axis of (3.6): (a) for vector; (b) for scalar gluons, with the cut-offs discussed in the text.

using the cut-off $Q_0^2 = 10\ \mathrm{GeV}^2$, and the c.m. energies $Q = 6, 8, 16$ and 32 GeV [σ_{pt} was defined in eq. (2.19)]. Motivated by eq. (2.24), we have taken $\alpha_V = 3\pi/28$, which lies within the preferred range, and arbitrarily set $\alpha_S = \alpha_V$. For comparison, we have also estimated the quantity (3.8) for conventional two-jet events, assuming a Poisson multiplicity distribution and an inclusive transverse momentum cross section

$$\frac{d\sigma}{dp_T} = 6\,\langle n_{ch}\rangle\, e^{-6p_\perp} \tag{3.9}$$

which has the same shape as that found in ISR pp collisions for $p_T \leqslant 1$ GeV. We observe that the bremsstrahlung process is negligible compared to the two-jet process for $Q = 6$ GeV. The gluonic bremsstrahlung contribution to $N(\tau)$ is no longer negligible when $Q = 8$ GeV: the structure of its contributions to $N(\tau)$ is different from that of two-jet events, where a large number of particles conspire to inflate τ. Gluonic contributions to $N(\tau)$ are dominant for $Q = 16, 32$ GeV.

The reason for the large values of $N(\tau)$ is of course that asymptotically we have the simple dimensional scaling law

$$\frac{d\sigma}{dx_T} = \frac{1}{Q^2} F(x_T)\,, \tag{3.10}$$

where $x_T \equiv 2p_T/Q$, for the large transverse momentum cross section. The scaling law (3.10) is only expected to be valid to within logarithms, coming possibly, for example, from the behaviour

$$\alpha_V(Q^2) \sim \frac{1}{\ln Q^2}$$

expected from asymptotic freedom. Using the cross sections (2.24) and (2.27) and the fragmentation function (3.7), we have calculated $1/\sigma_{pt}(d\sigma/dx_T)$, where again the momentum is measured relative to the axis minimizing $\Sigma_a p_T^{a2}$. The results are plotted in fig. 10 for subasymptotic energies, and again we observe that in a vector gluon theory deviations from a simple two-jet model (3.9) are only noticeable for $Q \geqslant 8$ GeV, and substantial for $Q \geqslant 16$ GeV. We have only plotted the cross sections for regions of x_T where the results are insensitive to quark or gluon fragmentation into low momentum hadrons. If we refer back to fig. 8, we see that the fragmentation function (3.7) is inapplicable for hadron energies $\lesssim 0(500)$ MeV. In the graphs of fig. 10 hadrons with $E \lesssim 800$ MeV have been excluded.

If the significant large p_T signal that we expect turns out to exist, it may be useful to analyze these events in a different way so as to see if there are three-jet structures. Present ideas about quark/gluon metamorphosis into hadrons suggest a third jet should exist in the direction of the large p_T particle. This prejudice is comforted by the observed [7] jet structure in large p_T pp collisions. Thus the oblate cigar or asymmetric frisbee of previous paragraphs is expected to manifest itself as an asymmetric "Y" or Mercedes-Benz symbol (see fig. 4) when there is enough phase space, i.e. Q

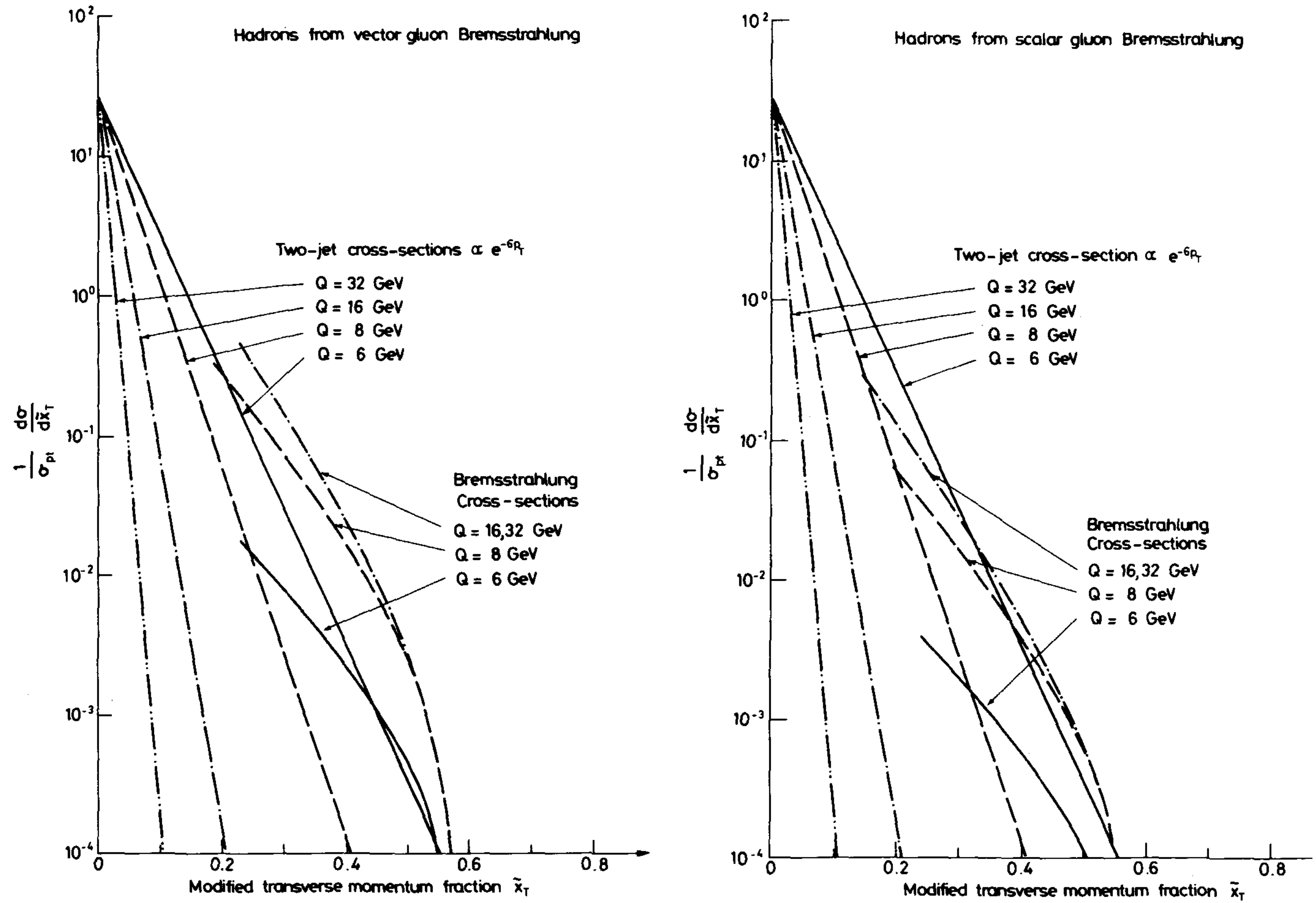

Fig. 11. The same cross sections relative to the second jet axis (3.11) (a) for vector and (b) for scalar gluons, with the cut-offs discussed in the text.

is high enough. It should be emphasized that as long as single hard gluon bremsstrahlung dominates large p_T, there should be no "opposite side" jet like that found [7] in pp collisions. Instead, transverse momentum is compensated non-locally, by a deflection in the directions of the basic quark and antiquark jets. This difference will be an important test of the difference between the dynamical pictures used for e^+e^- and pp collisions. In candidates for three-jet events it may be useful to divide the final state hadrons into two high energy jets travelling into opposite hemispheres, and a "shoulder" or jet of particles moving out at large p_T.

The natural definition of large p_T is then to take the transverse projection of particles in the third jet or "shoulder" relative to the less energetic of the two primary jets. Thus we define

$$\widetilde{p}_T = |xE_i \sin(\phi_i - \phi_j)| , \tag{3.11}$$

if i and j denote the least and medium energetic jets respectively. The inclusive cross section $d\sigma/d\widetilde{x}_T$ is plotted for vector gluons in fig. 11. Again we see observable deviations from a naive two-jet picture (3.9) when $Q \geqslant 8$ GeV.

Before leaving this phenomenological section, we should comment on the sensitivity of our results to changes in our assumptions. It is not clear whether our α_V should be identified with the asymptotic freedom running coupling constant, and if so at what value of the (momentum)^2 ? Q^2 ? s_{13} ? s_{23} ? However, we have checked that the cross sections of figs. 9–11 are decreased by $\leqslant 20\%$ at large p_T if we take $\alpha_V(Q_0^2 \simeq 10\ \mathrm{GeV}^2) = 3\pi/28$ and replace α_V by the effective coupling strength:

$$\alpha_V^{\mathrm{eff}} = \frac{3\pi}{28} \sqrt{\frac{\ln^2 (Q_0^2/\mu^2)}{\ln(s_{13}/\mu^2) \ln(s_{23}/\mu^2)}} , \qquad \mu^2 \simeq 1 , \tag{3.12}$$

in the double differential cross section (2.19). Because of the logarithms in eqs. (2.26) and (2.27) asymptotically free perturbation theory is unreliable for estimating the cross sections at high $Q = 16$ or 32 GeV, and moderate x_T or $\widetilde{x}_T (\leqslant 0.25?)$. In this region we expect the data to interpolate in an uncalculable but reasonably smooth way between the soft peak at small p_T and the hard bremsstrahlung contribution at large p_T. For this reason our cross-section estimates in this region are somewhat ballpark. This is also why we have not given estimates of the rise in $\langle p_T \rangle$ relative to the two-jet axis, though we expect [3] it to rise as Q (to within logarithms) when Q is sufficiently large.

Also potentially serious is the unreliability of the fragmentation function (3.7) chosen for gluons. Indeed, looking at fig. 4c one might naively expect more hadrons to be produced in gluon fragmentation than in quark fragmentation, and therefore that $f(x)$ for gluons should be more concentrated at low x. We have no control over this possibility: we can only argue that the effects we discuss are so gross that they could be seen even at $Q = 8$ GeV if $f(x)$ were reduced by a factor 2 for $x \geqslant \frac{1}{2}$.

4. Discussion

We have argued in this paper that at presently and shortly to be accessible e^+e^- energies, the dominant source of large p_T hadrons should be hard gluon bremsstrahlung [3]. Our calculations indicate that this effect may just be visible at SPEAR or DORIS energies $Q \approx 8$ GeV, but should be very important at PETRA and PEP energies. The first observable effect should be a tendency for the two-jet cigars to be unexpectedly oblate, with a high large p_T cross section. Eventually, events with large p_T would have a three-jet structure, without local compensation of p_T. Thus we may in principle have an observable experimental manifestation of the hitherto elusive gluons. Unfortunately, the gluonic jets are probably not very characteristic *, as they have zero net charge, strangeness, baryon number, charm, etc.

Given the size of the effects, one might wonder whether the picture may be more complicated: for example, is multiple gluon bremsstrahlung negligible? We have not calculated this, but the arguments of Sterman and of our formulae (2.19) and (2.23) suggest that the cross sections for multiple jets separated by finite angles fall by successive powers of $1/\ln Q^2$ in the $Q^2 \to \infty$ limit. Thus because of the smallness of the asymptotically free coupling constant, while we should expect more jets to exist at higher energies, they would probably be very difficult to observe.

One might also ask whether hard gluon bremsstrahlung could be significant in other processes. A very similar quark-gluon graph would be relevant in deep inelastic ep, μp, νp scattering. For large enough Q^2 we would expect events with two jets in the current fragmentation region, with properties similar to those discussed here. However, the lower values of Q^2 generally accessible in deep inelastic scattering compared with e^+e^- annihilation suggest that such structures may be more difficult to see in these reactions. Gluon bremsstrahlung may also be a noticeable correction to the two-jet structure found [7] in large p_T pp collisions at the ISR. One of the constituents (quarks, gluons ?) participating in the hard scattering may emit a bremsstrahlung gluon. Such effects are neither suggested nor excluded by present data: if they exist they may serve to distinguish between different models [4,5] for large p_T events in pp collisions.

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* For a contrary view, see ref. [15]. We are indebted to D.J. Broadhurst for bringing this reference to our attention.

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